

### 8.3.4 Lesson

Date: \_\_\_\_\_

### Fundamental Theorem of Similarity (FTS)

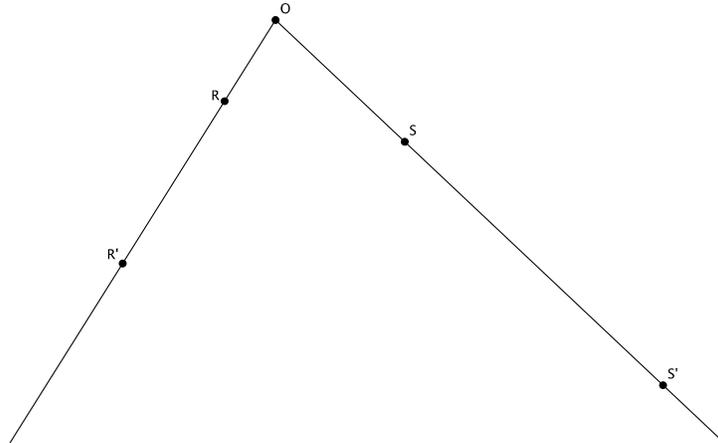
#### Student Objective

- I can experimentally verify the properties related to the Fundamental Theorem of Similarity (FTS).

#### Classwork

#### Exercises

1. In the diagram below, points  $R$  and  $S$  have been dilated from center  $O$ , by a scale factor of  $r = 3$ .



- If  $m\overline{OR} = 2.3$  cm, what is the length of  $\overline{OR'}$ ?
- If  $m\overline{OS} = 3.5$  cm, what is the length of  $\overline{OS'}$ ?
- Connect the point  $R$  to the point  $S$  and the point  $R'$  to the point  $S'$ . What do you know about  $\overline{RS}$  and  $\overline{R'S'}$ ?
- What is the relationship between the length of  $\overline{RS}$  and the length of  $\overline{R'S'}$ ?
- Identify pairs of angles that are equal in measure. How do you know they are equal?

## Example

1. Use a piece of notebook paper to verify the Fundamental Theorem of Similarity for a scale factor  $r$  that is  $r > 1$ .

**Check each circle as you have completed the direction.**

- Mark a point  $O$  on the first line of notebook paper.
- Draw a ray,  $\overrightarrow{OP}$ . Mark the point  $P$  on a line, several lines down from the center. Mark the point  $P'$  on the ray, and on a line of the notebook paper, farther from  $O$  than you placed point  $P$ . This ensures that you have a scale factor that is  $r > 1$ . Write your scale factor at the top of the notebook paper.
- Draw another ray,  $\overrightarrow{OQ}$ , and mark the points  $Q$  and  $Q'$  according to your scale factor.
- Connect points  $P$  and  $Q$ . Then, connect points  $P'$  and  $Q'$ .
- Place a point  $A$  on  $\overrightarrow{PQ}$  between points  $P$  and  $Q$ . Draw ray  $\overrightarrow{OA}$ . Mark the point  $A'$  at the intersection of  $\overrightarrow{P'Q'}$  and ray  $\overrightarrow{OA}$ .

Are  $\overrightarrow{PQ}$  and  $\overrightarrow{P'Q'}$  parallel lines? How do you know?

2.

- a. Which, if any, of the following pairs of angles are equal? Explain.
  - i.  $\angle OPQ$  and  $\angle OP'Q'$
  - ii.  $\angle OAQ$  and  $\angle OA'Q'$
  - iii.  $\angle OAP$  and  $\angle OA'P'$
  - iv.  $\angle OQP$  and  $\angle OQ'P'$
- b. Which, if any, of the following statements are true? Use a ruler and show your work to verify or dispute each statement.
  - i.  $m\overline{OP'} = r * m\overline{OP}$
  - ii.  $m\overline{OQ'} = r * m\overline{OQ}$
  - iii.  $m\overline{P'A'} = r * m\overline{PA}$
  - iv.  $m\overline{A'Q'} = r * m\overline{AQ}$
- c. Do you believe that the Fundamental Theorem of Similarity (FTS) is true when the scale factor is  $r > 1$ . Explain.

### Lesson Summary

**Fundamental Theorem of Similarity:** Given a dilation with center  $O$  and scale factor  $r$ , then for any two points  $P$  and  $Q$  in the plane so that  $O$ ,  $P$ , and  $Q$  are not collinear,  $\overrightarrow{PQ}$  and  $\overrightarrow{P'Q'}$  are parallel, where  $P' = \text{dilation}(P)$  and  $Q' = \text{dilation}(Q)$ , and furthermore,  $m\overline{P'Q'} = r * m\overline{PQ}$ .

**Homework****Homework****Homework****Homework****Homework**

1. Use a piece of notebook paper to verify the Fundamental Theorem of Similarity for a scale factor  $r$  that is  $0 < r < 1$ .

**Check each circle as you have completed the direction.**

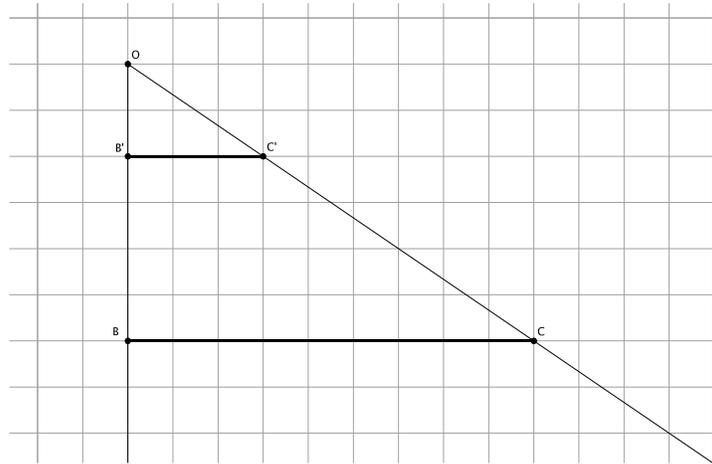
- Mark a point  $O$  on the first line of notebook paper.
- Draw a ray,  $\overrightarrow{OP}$ . Mark the point  $P$  on a line, several lines down from the center. Mark the point  $P'$  on the ray, and on a line of the notebook paper, closer to  $O$  than you placed point  $P$ . This ensures that you have a scale factor that is  $0 < r < 1$ . Write your scale factor at the top of the notebook paper.
- Draw another ray,  $\overrightarrow{OQ}$ , and mark the points  $Q$  and  $Q'$  according to your scale factor.
- Connect points  $P$  and  $Q$ . Then, connect points  $P'$  and  $Q'$ .
- Place a point  $A$  on  $\overrightarrow{PQ}$  between points  $P$  and  $Q$ . Draw ray  $\overrightarrow{OA}$ . Mark the point  $A'$  at the intersection of  $\overrightarrow{P'Q'}$  and ray  $\overrightarrow{OA}$ .

Are  $\overrightarrow{PQ}$  and  $\overrightarrow{P'Q'}$  parallel lines? How do you know?

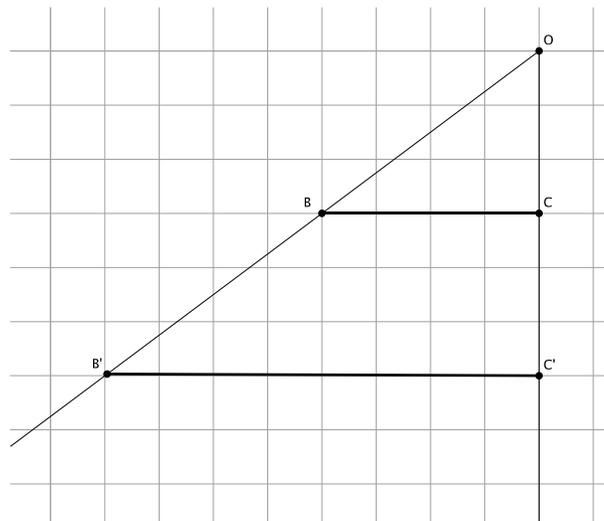
2.

- a. Which, if any, of the following pairs of angles are equal? Explain.
- i.  $\angle OPQ$  and  $\angle OP'Q'$
  - ii.  $\angle OAQ$  and  $\angle OA'Q'$
  - iii.  $\angle OAP$  and  $\angle OA'P'$
  - iv.  $\angle OQP$  and  $\angle OQ'P'$
- b. Which, if any, of the following statements are true? Use a ruler and show your work to verify or dispute each statement.
- i.  $m\overline{OP'} = r * m\overline{OP}$
  - ii.  $m\overline{OQ'} = r * m\overline{OQ}$
  - iii.  $m\overline{P'A'} = r * m\overline{PA}$
  - iv.  $m\overline{A'Q'} = r * m\overline{AQ}$
- c. Do you believe that the Fundamental Theorem of Similarity (FTS) is true even when the scale factor is  $0 < r < 1$ . Explain.

3. Caleb sketched the following diagram on graph paper. He dilated points  $B$  and  $C$  from center  $O$ .



- What is the scale factor  $r$ ? Show your work.
  - Verify the scale factor with a different set of segments.
  - Which segments are parallel? How do you know?
  - Which angles are equal in measure? How do you know?
3. Points  $B$  and  $C$  were dilated from center  $O$ .



- What is the scale factor  $r$ ? Show your work.
- If  $m\overline{OB} = 5$ , what is  $m\overline{OB'}$ ?
- How does the perimeter of  $\triangle OBC$  compare to the perimeter of  $\triangle OB'C'$ ?
- Did the perimeter of  $\triangle OB'C'$  equal  $r \times$  the perimeter of  $\triangle OBC$ ? Explain.