

## Sequencing Translations and Reflections

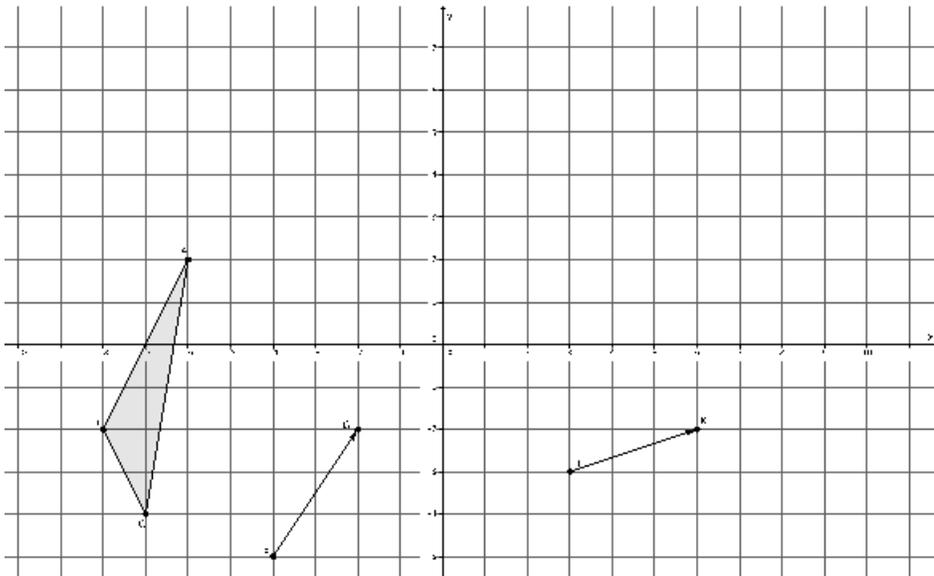
## Student Objectives

- I know that a sequence of transformations (one move on the plane followed by another) enjoys the same properties as a single translation with respect to lengths of segments and degrees of angles.
- I know that a translation along a vector followed by another translation along the same vector in the opposite direction can move all points of a plane back to its original position.
- I know that the reflection is its own inverse transformation.
- I understand that a sequence of a reflection followed by a translation is not equal to a translation followed by a reflection.

## Classwork

## Exploratory Challenge

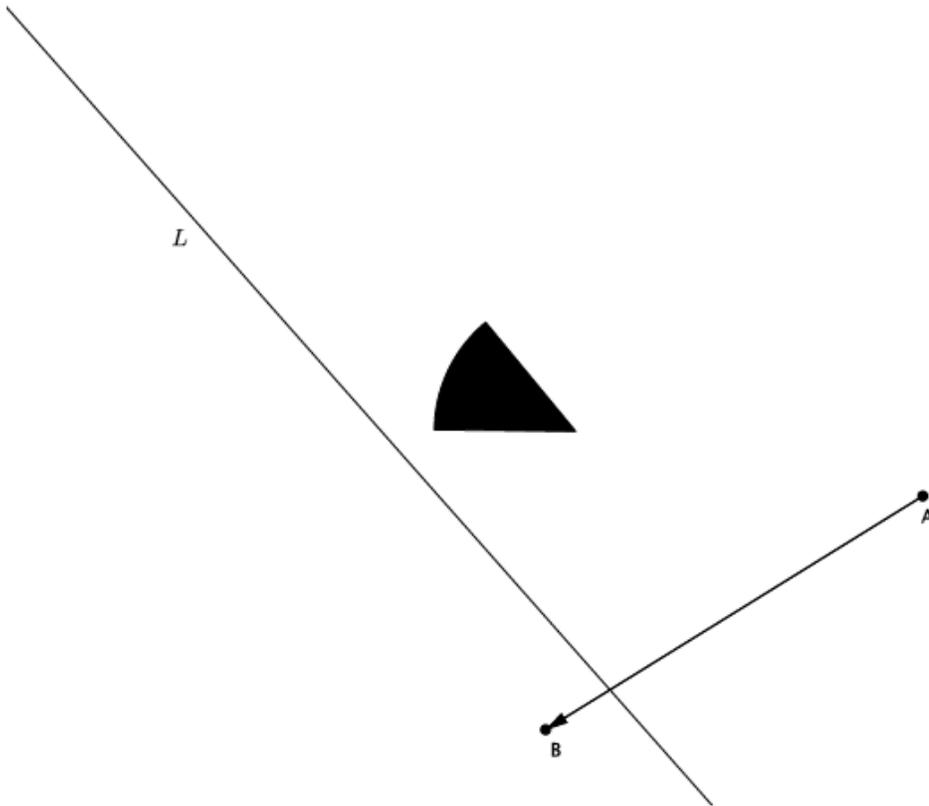
1. Translate  $\triangle ABC$  along vector  $\overrightarrow{FG}$  and then translate its image along vector  $\overrightarrow{JK}$ . Be sure to label the images appropriately.



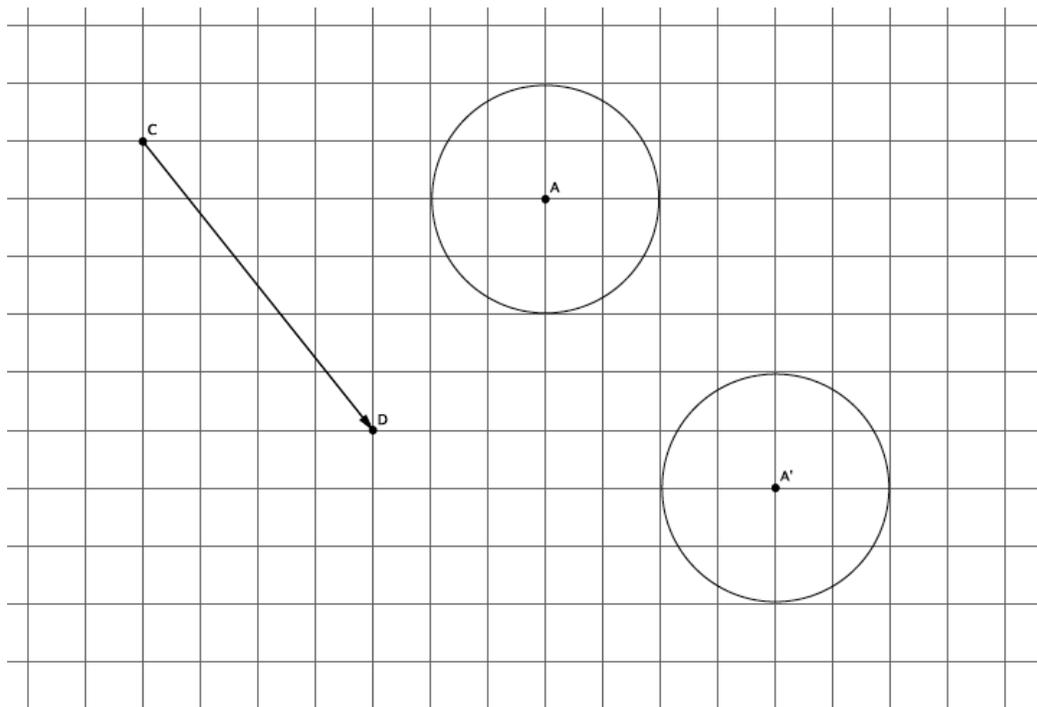
- a. How does the size of  $\angle ABC$  compare to the size of  $\angle A''B''C''$ ?
- b. How does the length of  $\overline{AB}$  compare to the length of  $\overline{A''B''}$ ?
- c. Why do you think what you observed in parts (a) and (b) were true?
- d. Where is the image of triangle  $ABC$  if you first translate along vector  $\overrightarrow{JK}$  and then along vector  $\overrightarrow{FG}$ ? This shows that sequences of translations are \_\_\_\_\_.

Let the black figure below be figure  $S$ .

2. If figure  $S$  was translated along vector  $\overrightarrow{AB}$  resulting in  $Translation(Figure S)$ , what transformation would map  $Translation(Figure S)$  back onto its original position?
3. If figure  $S$  was reflected across line  $L$  resulting in  $Reflection(Figure S)$ , what transformation would map  $Reflection(Figure S)$  back onto its original position?
4. Use a transparency to perform the following sequence: Translate figure  $S$  along vector  $\overrightarrow{AB}$ , then reflect figure  $S$  across line  $L$ . Label the image  $S'$ .
5. Use a transparency to perform the following sequence: Reflect figure  $S$  across line  $L$ , then translate figure  $S$  along vector  $\overrightarrow{AB}$ . Label the image  $S''$ .
6. Are figures  $S$  and  $S'$  in the same location? This shows that sequences of translations and reflections are \_\_\_\_\_.



7. The picture below shows the translation of Circle A along vector  $\overrightarrow{CD}$ . Name the vector that will map the image of Circle A back onto itself. Translation along vector \_\_\_\_\_ is the inverse transformation of a translation along vector  $\overrightarrow{CD}$ .

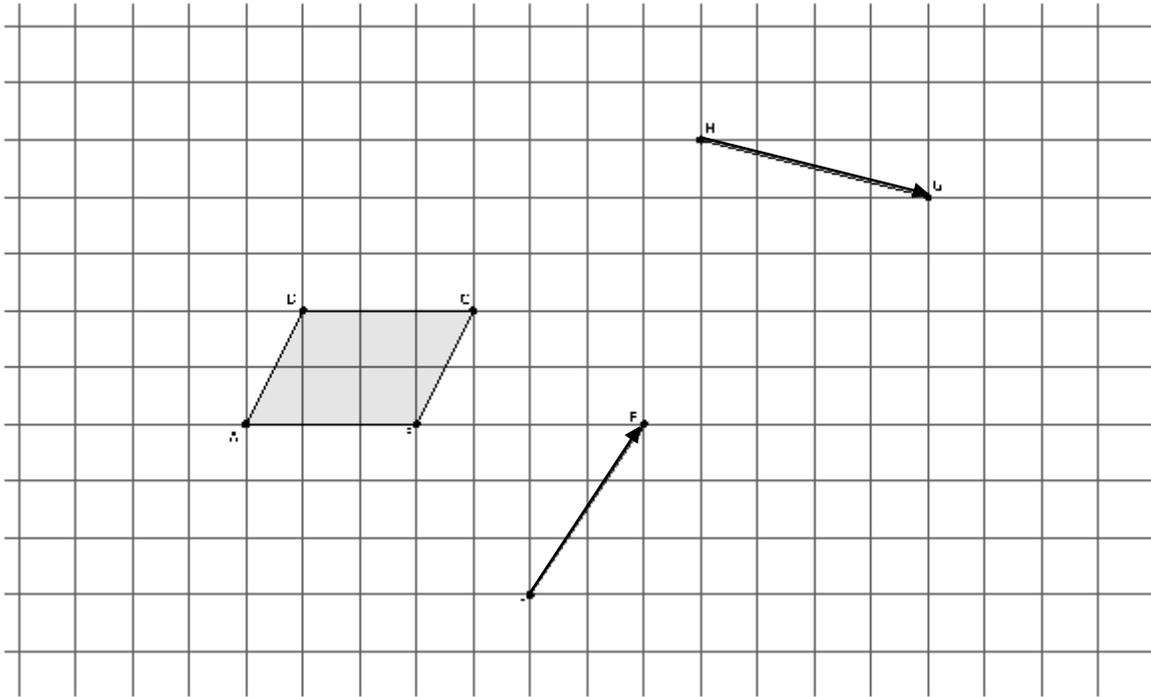


### Lesson Summary

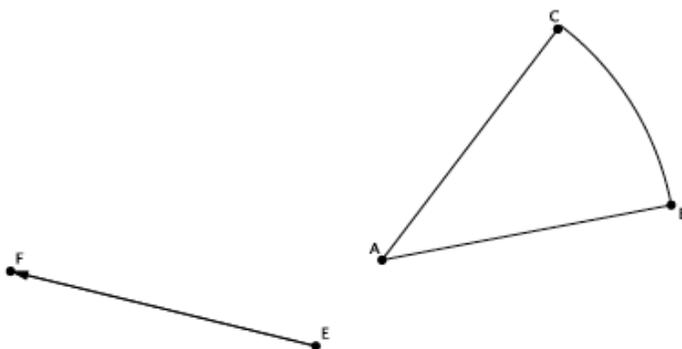
- Translating a figure along one vector then translating its image along another vector is an example of a sequence of transformations.
- A sequence of translations enjoys the same properties as a single translation. Specifically, the figures' lengths and degrees of angles are preserved.
- If a figure undergoes two transformations,  $F$  and  $G$ , and is in the same place it was originally, then the figure has been mapped onto itself.
- A reflection across a line followed by a reflection across the same line places all figures in the plane back onto their original position.
- A reflection followed by a translation does not place a figure in the same location in the plane as a translation followed by a reflection. The order in which we perform a sequence of rigid motions matters.

**Homework Homework Homework Homework Homework**

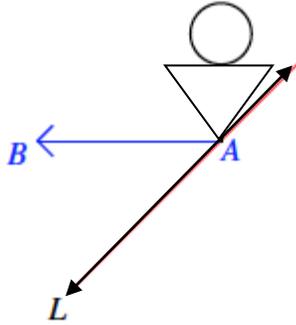
1. Perform the following sequence translations on Parallelogram  $ABCD$  (a quadrilateral in which both pairs of opposite sides are parallel) a translation along vector  $\vec{HG}$  and then a translation along vector  $\vec{EF}$ . Label the translated images.



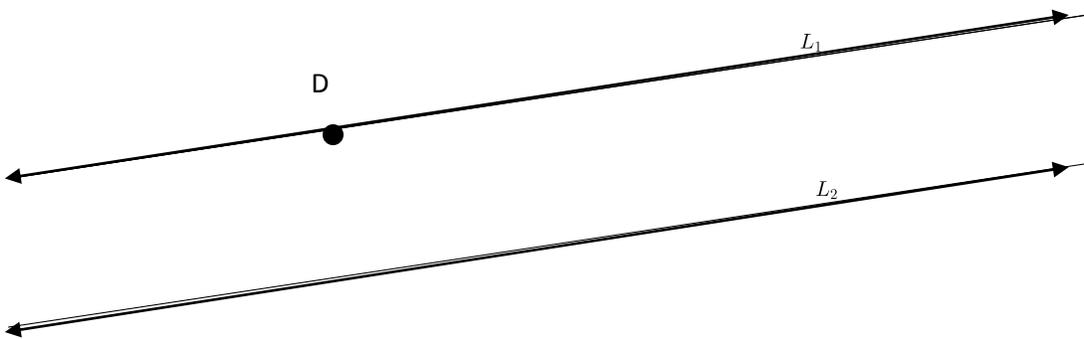
2. What do you know about  $\overline{AD}$  and  $\overline{BC}$  compared with  $\overline{A'D'}$  and  $\overline{B'C'}$ ? Explain.
3. Are  $\overline{A'B'}$  and  $\overline{A''B''}$  equal in length? How do you know?
4. Translate the curved shape  $ABC$  along the given vector. Label the image.
5. What vector would map the shape  $A'B'C'$  back onto shape  $ABC$ ?



6. Let there be a reflection across line  $L$  and let there be a translation along vector  $\overrightarrow{AB}$  as shown. If  $S$  denotes the figure below (circle and triangle), compare the translation of  $S$  followed by the reflection of  $S$  with the reflection of  $S$  followed by the translation of  $S$ .



7. Let  $L_1$  and  $L_2$  be parallel lines and let  $Reflection_1$  and  $Reflection_2$  be the reflections across  $L_1$  and  $L_2$ , respectively (in that order). Show that a  $Reflection_2$  followed by  $Reflection_1$  is not equal to a  $Reflection_1$  followed by  $Reflection_2$ . (Hint: Take a point on  $L_1$  and see what each of the sequences does to it.)



8. Name another transformation(s) that gives you the same image of point  $D$  as the sequence of transformations of  $Reflection_1$  and  $Reflection_2$ .