Math 7 Period__________ Name____________________

7.6 Expressions, Equations, and Inequalities Date___________________

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## OVERVIEW

In this unit, students solve equations of the forms px+q=r and p(x+q)=r where p, q, and r are rational numbers. They draw, interpret, and write equations in one variable for balanced “hanger diagrams,” and write expressions for sequences of instructions, e.g., “number puzzles.” They use tape diagrams together with equations to represent situations with one unknown quantity. They learn algebraic methods for solving equations. Students solve linear inequalities in one variable and represent their solutions on the number line. They understand and use the terms “less than or equal to” and “greater than or equal to,” and the corresponding symbols. They generate expressions that are equivalent to a given numerical or linear expression. Students formulate and solve linear equations and inequalities that represent real-world situations.

CHALLENGE 78
I can find and use the OPPOSITE of a number. RECIPROCAL of a number.

7.6.0 Algebra Vocabulary Date_________________________

**Student Objectives:**

- I can identify how many terms an expression has and what the coefficients, constants, and like terms of that expression are.
- I recognize the properties of operations.

## 0.1

| Vocabulary | Definition | Example |
|---|---|---|
| Expression | A mathematical statement expressing something, can be a number or variable, or have an operation or operations with variables and/or numbers | |
| Equivalent Expressions | Expressions that have the same value, no matter what the value of a variable | |
| Equation | A mathematical sentence containing an equal sign | |
| Terms | Each part of an expression separated by addition or subtraction | |
| Coefficient | The number that multiplies a variable | |
| Like Terms | Terms with the same variable raised to the same exponent | |
| Constant | A term without a variable | |
| Simplest Form/ Standard Form | An algebraic expression without parentheses and no like terms | |

**0.2** Fill in the chart for each of the following expressions

| Expression | Number of Terms | Like Terms | Coefficients | Constants |
|---|---|---|---|---|
| 1. $3a + 6 + 5$ | | | | |
| 2. $8b + 7 - 4b - 3$ | | | | |
| 3. $5c - 4c + c - 1$ | | | | |

**0.3** - Fill in the chart for each of the following expressions

| Expression | Number of Terms | Like Terms | Coefficients | Constants |
|---|---|---|---|---|
| 1. $3a + 6 + 5a - 2$ | | | | |
| 2. $8b - 8b + 3 - 3$ | | | | |
| 3. $5c - c + 1$ | | | | |

## 0.4 Properties of Operations:

| | |
|---|---|
| Commutative Property of Addition | $x + 2 = 2 + x$ |
| Associative Property of Addition | $(8 + 7) + 3 = 8 + (7 + 3)$ |
| Identity Property of Addition | $0 + 7 = 7$ |
| Commutative Property of Multiplication | $2x = x \cdot 2$ |
| Associative Property of Multiplication | $2(5x) = (2 \cdot 5)x$ |
| Identity Property of Multiplication | $1x = x$ |
| Distributive Property of Multiplication over Addition or Subtraction | $5(x + 1) = 5x + 5$ |

Identify the property shown in each of the following examples.

1. $-5(3x-2) = -15x+10$ ________________________________________________
2. $1m = m$ ________________________________________________
3. $7+(c-2) = (c-2)+7$ ________________________________________________
4. $b = b+3-3$ ________________________________________________
5. $3(6c) = (3 \cdot 6)c$ ________________________________________________
6. $(x + 5) + 3 = x + (5 + 3)$ ________________________________________________
7. $(4 + g) \cdot 2 = 2(4 + g)$ ________________________________________________

7.6.1A Combining Like Terms (Part 1)                    Date______________________

**Student Objective:** Let's see how we can tell that expressions are equivalent.

## 1A.1: Why is it True?

Explain why each statement is true.

1. $5 + 2 + 3 = 5 + (2 + 3)$

2. $9a$ is equivalent to $11a - 2a$.

3. $7a + 4 - 2a$ is equivalent to $7a + -2a + 4$.

4. $8a - (8a - 8)$ is equivalent to 8.

## 1A.2: Bags of Gold and Coins

Diego and Jada are both trying to write an expression with fewer terms that is equivalent to $4b + 4 + 3b + 2$

- Jada thinks $7b + 6$ is equivalent to the original expression.
- Diego thinks $13a$ is equivalent to the original expression.

We can show expressions are equivalent by thinking about what variables and constants represent. We know that a variable represents an unknown number. We are going to let b = the number of gold coins in a bag and constants represent the number of loose coins that you can see.

______________________ is correct because 4 bags of gold and 4 coins plus 3 bags of gold and 2 coins is ____ bags of gold and ____ coins.

### 1A.3: Properties and Order of Operations

We know that $4x + 2 = 2 + 4x$ because of the ______________________Property of Addition.

We can use number examples and the order of operations to show that we can not combine (add or subtract) terms that are not like terms. Let's see what happens when $x = 10$.

Do we get the same value for $6x$?_________

### 1A.4: Combining Like Terms

Examples 1 – Write the following expressions in simplest form.

a) $3a + 6 + 5a - 2$ b) $8b + 8 - 4b - 3$ c) $5c - 4c + c - 3c$

Example 2 – Why isn't $7m + 4$ equal to $11m$?

### 1A.5: Combining Like Terms

Exercises 1 – Write the following expressions in simplest form.

d) $5x + 2 + 5x + 3$ e) $5y + 2 - 2y - 3$ f) $5z + 12 - 5z - 3$

### 1A.6: Making Sides Equal

Replace each ( ) with an expression that will make the left side of the equation equivalent to the right side.

1. $6x + (\quad) = 10x$
2. $6x + (\quad) = 2x$
3. $6x + (\quad) = -10x$
4. $6x + (\quad) = 0$
5. $6x + (\qquad) = 10$

### Lesson 1A Summary

There are many ways to write equivalent expressions that may look very different from each other. We have several tools to find out if two expressions are equivalent.

- Two expressions are definitely not equivalent if they have different values when we substitute the same number for the variable. For example, $2(-3 + x) + 8$ and $2x + 5$ are not equivalent because when $x$ is 1, the first expression equals 4 and the second expression equals 7.
- If two expressions are equal for many different values we substitute for the variable, then the expressions *may* be equivalent, but we don't know for sure. It is impossible to compare the two expressions for all values. To know for sure, we use properties of operations.

7.6.1B Simplifying and Factoring  Date______________________

**Student Objective:** Let's use the distributive property to write expressions in different ways.

**1B.1: Number Talk: Parentheses** Find the value of each expression mentally.

$2 + 3 \cdot 4$  $2 - 3 \cdot 4$

$(2 + 3)(4)$  $2 - (3 + 4)$

**1B.2:** Rewrite the expressions as a product of two factors or in standard form.

| Factored Form | Standard Form |
|---|---|
| a. $2(x + 5)$ | |
| b. $3(x + 4)$ | |
| c. $6(x + 1)$ | |
| d. $7(x - 3)$ | |
| e. | $5x + 30$ |
| f. | $8x + 8$ |
| g. | $3x - 12$ |
| h. | $15x + 20$ |

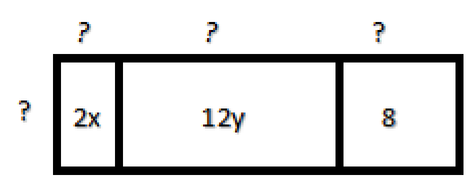
**1B.3: Write the product and sum of the expressions being represented in the rectangular array.**

Notes:

**To factor an expression**:

1. Find the ________ of the terms.
2. Write the GCF and parentheses.
3. Write the **factors** in the parentheses that you need to multiply back to the original expression.

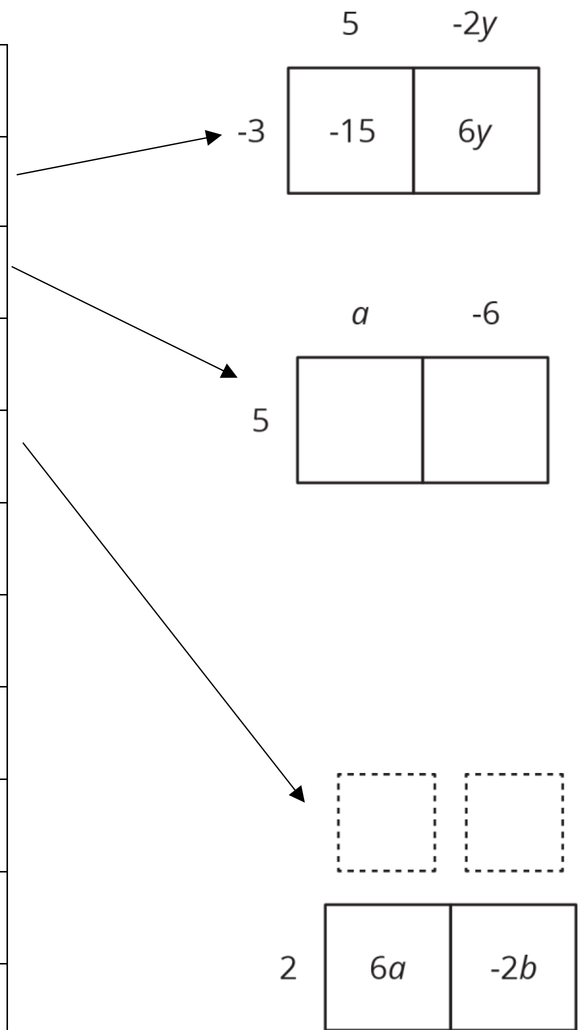
Product  Sum



### 1B.4: Factoring and Simplifying with Negative Numbers

In each row, write the equivalent expression. If you get stuck, use a diagram to organize your work. The first row is provided as an example. Diagrams are provided for the first three rows.

| **Factored** | **Simplified** |
|---|---|
| $-3(5-2y)$ | $-15+6y$ |
| $5(a-6)$ | |
| $-(2x-3y)$ | |
| | $6a-2b$ |
| $7(2n+7)$ | |
| | $10x-20y+15$ |
| | $9a-6b+3$ |
| $4(2w+5)$ | |
| $5(2x-3y)$ | |
| $-5(2x-3y)$ | |

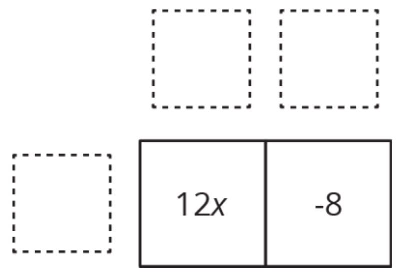


## Lesson 1B Summary

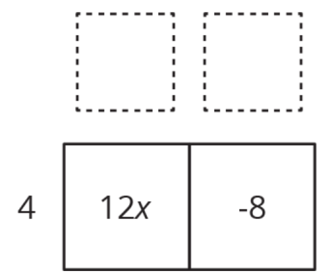
We can use properties of operations in different ways to rewrite expressions and create equivalent expressions. We have already seen that we can use the distributive property to simplify an expression, for example $3(x + 5) = 3x + 15$. We can also use the distributive property in the other direction and factor an expression, for example $8x + 12 = 4(2x + 3)$.

We can organize the work of using distributive property to rewrite the expression $12x - 8$. In this case we know the product and need to find the factors.

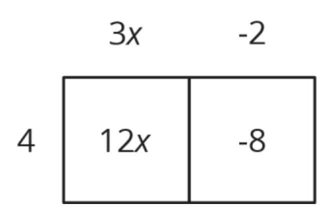
The terms of the product go inside:




We look at the expressions and think about a factor they have in common. $12x$ and $-8$ each have a factor of 4. We place the common factor on one side of the large rectangle:




Now we think: "4 times *what* is $12x$?" "4 times *what* is -8?" and write the other factors on the other side of the rectangle:




So, $12x - 8$ is equivalent to $4(3x - 2)$.

7.6.1C Combining Like Terms (Part 2) Date______________________

**Student Objective:** Let's see how we can combine terms in an expression to write it with fewer terms.

### 1C.1: Are They Equal?

Select **all** expressions that are equal to $8 - 12 - (6 + 4)$.

a. $8 - 6 - 12 + 4$

b. $8 - 12 - 6 - 4$

c. $8 - 12 + (6 + 4)$

d. $8 - 12 - 6 + 4$

e. $8 - 4 - 12 - 6$

### 1C.2: Card Sort

Your teacher will give you a set of cards, which include the expressions in column A. Match each expression in column A with two equivalent expressions (within your cards). Write down which expressions you match with each expression in column A. Be prepared to explain your reasoning.

**A**

| | | |
|---|---|---|
| a. $(9x + 5y) + (3x + 7y)$ | | |
| b. $(9x + 5y) - (3x + 7y)$ | | |
| c. $(9x + 5y) - (3x - 7y)$ | | |
| d. $9x - 7y + 3x + 5y$ | | |
| e. $9x - 7y + 3x - 5y$ | | |
| f. $9x - 7y - 3x - 5y$ | | |

### 1C.3: Seeing Structure and Factoring

Write each expression with fewer terms. Show or explain your reasoning.

1. $3(x + 4) + 15$

2. $3x + 4x - 5x$

3. $3(x - 2) + 4(x - 2) - 5(x - 2)$

4. $3x + 4 + 15$

## Lesson 1C Summary

Combining like terms is a useful strategy that we will see again and again in our future work with mathematical expressions. It is helpful to review the things we have learned about this important concept.

- Combining like terms is an application of the distributive property. For example:

$$2x + 9x$$
$$(2 + 9) \cdot x$$
$$11x$$

- It often also involves the commutative and associative properties to change the order or grouping of addition. For example:

$$2a + 3b + 4a + 5b$$
$$2a + 4a + 3b + 5b$$
$$(2a + 4a) + (3b + 5b)$$
$$6a + 8b$$

- We can't change order or grouping when subtracting; so in order to apply the commutative or associative properties to expressions with subtraction, we need to rewrite subtraction as addition. For example:

$$2a - 3b - 4a - 5b$$
$$2a + -3b + -4a + -5b$$
$$2a + -4a + -3b + -5b$$
$$-2a + -8b$$
$$-2a - 8b$$

- Since combining like terms uses properties of operations, it results in expressions that are equivalent.

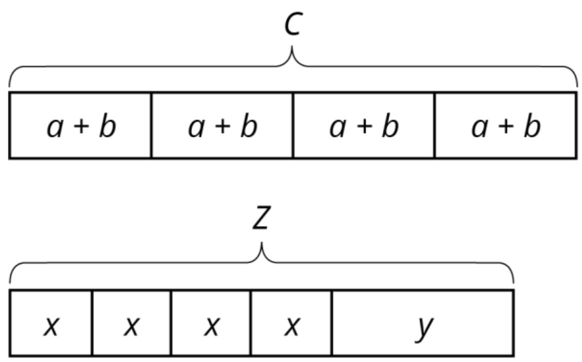
- The like terms that are combined do not have to be a single number or variable; they may be longer expressions as well. Terms can be combined in any sum where there is a common factor in all the terms. For example, each term in the expression $5(x + 3) - 0.5(x + 3) + 2(x + 3)$ has a factor of $(x + 3)$. We can rewrite the expression with fewer terms by using the distributive property:

$$5(x + 3) - 0.5(x + 3) + 2(x + 3)$$
$$(5 - 0.5 + 2)(x + 3)$$
$$6.5(x + 3)$$

7.6.2 Reasoning about Contexts with Tape Diagrams (Part 1)
Date______________________

**Student Objective**: Let’s use tape diagrams to write and solve equations.

## 2.1: Remembering Tape Diagrams



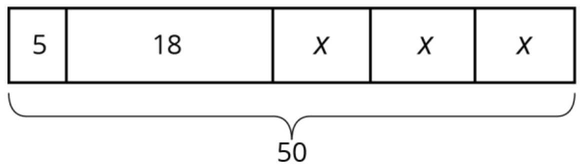

1. What do you notice? What do you wonder?

2. What are some possible values for $a$, $b$, and $c$ in the first diagram?
   For $x$, $y$, and $z$ in the second diagram? How did you decide on those values?

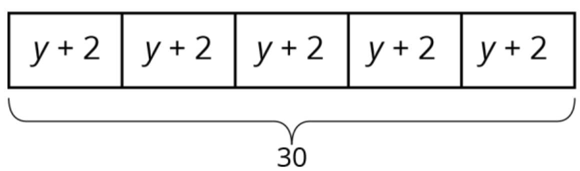
## 2.2: Every Picture Tells a Story

Here are three stories with a diagram that represents it. Write an equation that represents the diagram and find the solution for the unknown.

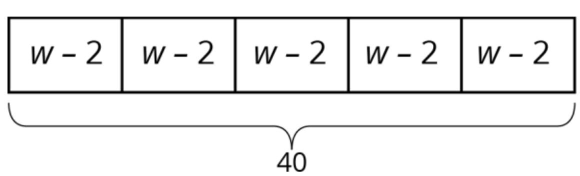
1. Mai made 50 flyers for five volunteers in her club to hang up around school. She gave 5 flyers to the first volunteer, 18 flyers to the second volunteer, and divided the remaining flyers equally among the three remaining volunteers.




2. To thank her five volunteers, Mai gave each of them the same number of stickers. Then she gave them each two more stickers. Altogether, she gave them a total of 30 stickers.




3. Mai distributed another group of flyers equally among the five volunteers. Then she remembered that she needed some flyers to give to teachers, so she took 2 flyers from each volunteer. Then, the volunteers had a total of 40 flyers to hang up.



## 2.3: Every Story Needs a Picture

Here are three more stories. Draw a tape diagram to represent each story. Then describe how you would find any unknown amounts in the stories.

1. Noah and his sister are making gift bags for a birthday party. Noah puts 3 pencil erasers in each bag. His sister puts $x$ stickers in each bag. After filling 4 bags, they have used a total of 44 items.

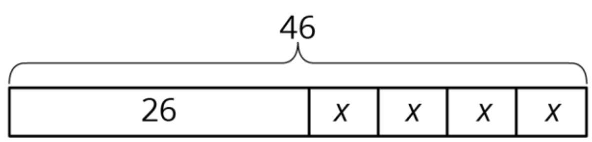
2. Noah's family also wants to blow up a total of 60 balloons for the party. Yesterday they blew up 24 balloons. Today they want to split the remaining balloons equally between four family members.

3. Noah's family bought some fruit bars to put in the gift bags. They bought one box each of four flavors: apple, strawberry, blueberry, and peach. The boxes all had the same number of bars. Noah wanted to taste the flavors and ate one bar from each box. There were 28 bars left for the gift bags.

## Lesson 2 Summary

Tape diagrams are useful for representing how quantities are related and can help us answer questions about a situation.

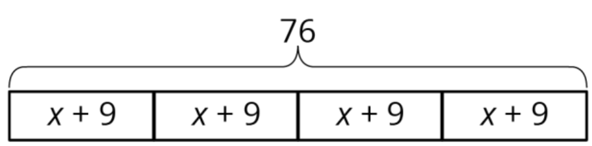
Suppose a school receives 46 copies of a popular book. The library takes 26 copies and the remainder are split evenly among 4 teachers. How many books does each teacher receive? This situation involves 4 equal parts and one other part. We can represent the situation with a rectangle labeled 26 (books given to the library) along with 4 equal-sized parts (books split among 4 teachers). We label the total, 46, to show how many the rectangle represents in all. We use a letter to show the unknown amount, which represents the number of books each teacher receives. Using the same letter, $x$, means that the same number is represented four times.




Some situations have parts that are all equal, but each part has been increased from an original amount:

A company manufactures a special type of sensor, and packs them in boxes of 4 for shipment. Then a new design increases the weight of each sensor by 9 grams. The new package of 4 sensors weighs 76 grams. How much did each sensor weigh originally?

We can describe this situation with a rectangle representing a total of 76 split into 4 equal parts. Each part shows that the new weight, $x + 9$, is 9 more than the original weight, $x$.



7.6.3 Tape Diagrams and Verifying Solutions Date________________________

**Student Objective:** Let's see how equations can describe tape diagrams. Let's check solutions to equations.

## 3.1: Find Equivalent Expressions

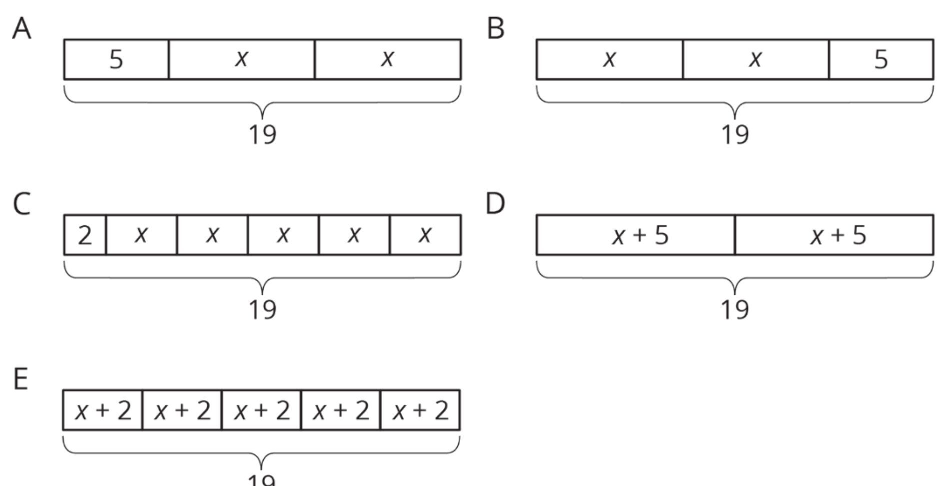
Select **all** the expressions that are equivalent to $7(2-3n)$. Explain how you know each expression you select is equivalent. Hint: there are 2 correct answers.

a. $9-10n$
b. $14-3n$
c. $14-21n$
d. $(2-3n)\cdot 7$
e. $7\cdot 2\cdot(-3n)$

## 3.2: Matching Equations to Tape Diagrams

Match each equation to one of the tape diagrams. Be prepared to explain how the equation matches the diagram.

- $2x+5=19$
- $2+5x=19$
- $2(x+5)=19$
- $5(x+2)=19$
- $19=5+2x$
- $(x+5)\cdot 2=19$
- $19=(x+2)\cdot 5$
- $19\div 2=x+5$
- $19-2=5x$




## 3.3: Checking Solutions to Equations

Check to see if $y=-3$ is a solution to the equation:

$$11y+4=3(y+18)$$

Check to see if $x = 2$ is a solution to the equation:

- $60 = 3x + 18$
- $60 = 3(x + 18)$

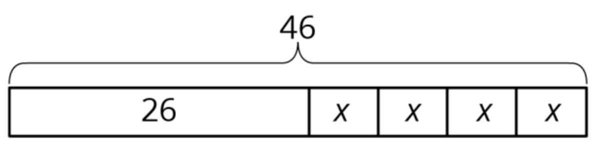
Check to see if $x = 10$ is a solution to the equation:

- $2 + 5x = 30$
- $2(x + 5) = 30$
- $5(x + 2) = 5x + 2$

## Lesson 3 Summary

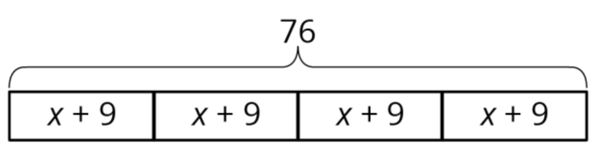
We have seen how tape diagrams represent relationships between quantities. Because of the meaning and properties of addition and multiplication, more than one equation can often be used to represent a single tape diagram.

Let's take a look at two tape diagrams.




We can describe this diagram with several different equations. Here are some of them:

- $26 + 4x = 46$, because the parts add up to the whole.
- $4x + 26 = 46$, because addition is commutative.
- $46 = 4x + 26$, because if two quantities are equal, it doesn't matter how we arrange them around the equal sign.
- $4x = 46 - 26$, because one part (the part made up of 4 $x$'s) is the difference between the whole and the other part.




For this diagram:

- $4(x + 9) = 76$, because multiplication means having multiple groups of the same size.
- $(x + 9) \cdot 4 = 76$, because multiplication is commutative.
- $76 \div 4 = x + 9$, because division tells us the size of each equal part

To determine if a number is a solution to an equation, substitute the number into the equation for the variable (letter) and check to see if the resulting number sentence is true. If it is true, then the number is a solution to the equation. For example, $7\frac{1}{2}$ is a solution to $2n = 15$ because $2\left(7\frac{1}{2}\right) = 15$.

7.6.4 Reasoning about Equations with Crates and Rocks
Date______________________

**Student Objective:** Let's use properties of equality to solve equations.

### 4.1: Algebra Talk: Seeing Structure

**Properties of Equality:**

- Addition Property of Equality If $\boldsymbol{A} = \boldsymbol{B}$, then $\boldsymbol{A} + \boldsymbol{C} = \boldsymbol{B} + \boldsymbol{C}$
- Subtraction Property of Equality If $\boldsymbol{A} = \boldsymbol{B}$, then $\boldsymbol{A} - \boldsymbol{C} = \boldsymbol{B} - \boldsymbol{C}$
- Multiplication Property of Equality If $\boldsymbol{A} = \boldsymbol{B}$, then $\boldsymbol{A} \cdot \boldsymbol{C} = \boldsymbol{B} \cdot \boldsymbol{C}$
- Division Property of Equality If $\boldsymbol{A} = \boldsymbol{B}$, then $\frac{\boldsymbol{A}}{\boldsymbol{C}} = \frac{\boldsymbol{B}}{\boldsymbol{C}}$, where $\boldsymbol{C}$ is not equal to zero

Procedure to **solve** equations:

1) Write each expression in **standard form** (simplest form)
2) Use the **addition** or **subtraction** property of equality
3) Write each expression in **standard form** (simplest form)
4) Use the **multiplication** or **division** property of equality
5) Write each expression in **standard form** (simplest form)
6) Check

evaluating/simplifying expressions ↓

G
E
MD →
AS →

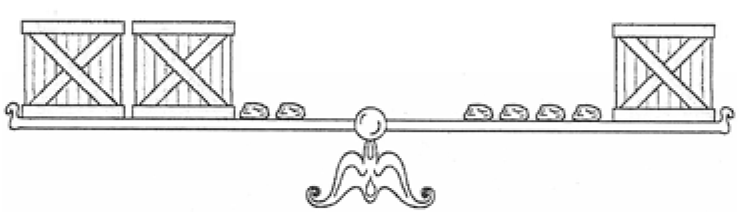
solving equations ↑

## Geology Rocks Equations

NAME ______________________

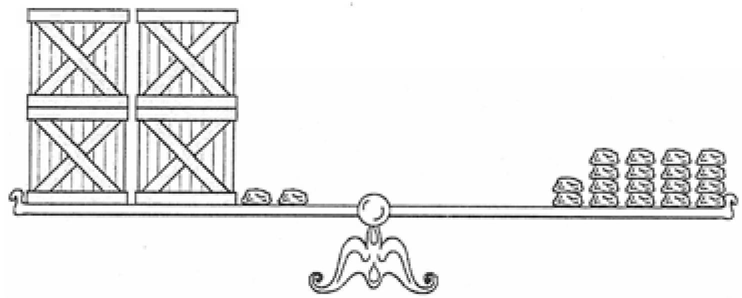
Mr. Anderson is a geologist and has a laboratory full of rocks. He knows that each rock weighs exactly one pound (+1), and he would like to figure out how many rocks are in each crate. To figure that out without opening the crates, Mr. Anderson places crates and rocks on a scale until they are balanced. Using his math skills, he is able to reason how many rocks are in each crate without having to look inside.

1. The following picture represents the first set of crates and rocks Mr. Anderson put on the balance. How many rocks are inside each crate?



Mr. Anderson has made several picture representations on his clipboard of other combinations of crates and rocks that balanced. Can you figure out how many rocks are in each set of crates?

2.



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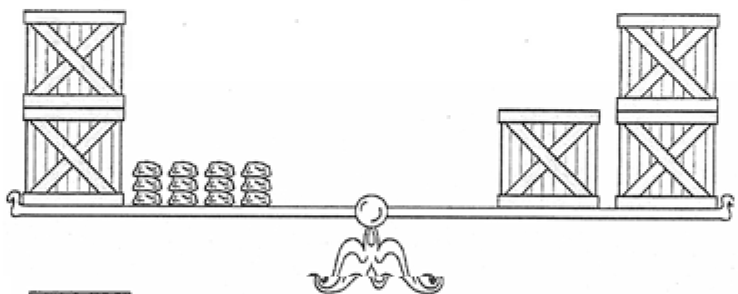

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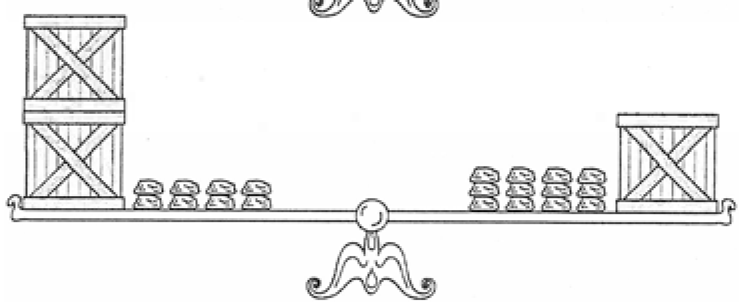


***Mr. Anderson wrote down the following equations but did not draw any pictures. Draw a picture and find the value of x using both the pictures and algebra.***

5. $14 = 4x + 6$

6. $2(x + 4) = 16$

## Lesson 4 Summary

Many situations can be represented by equations. Writing an equation to represent a situation can help us express how quantities in the situation are related to each other, and can help us reason about unknown quantities whose value we want to know. Here are three situations:

1. An architect is drafting plans for a new supermarket. There will be a space 144 inches long for rows of nested shopping carts. The first cart is 34 inches long and each nested cart adds another 10 inches. The architect want to know how many shopping carts will fit in each row.
2. A bakery buys a large bag of sugar that has 34 cups. They use 10 cups to make some cookies. Then they use the rest of the bag to make 144 giant muffins. Their customers want to know how much sugar is in each muffin.
3. Kiran is trying to save $144 to buy a new guitar. He has $34 and is going to save $10 a week from money he earns mowing lawns. He wants to know how many weeks it will take him to have enough money to buy the guitar.

We see the same three numbers in the situations: 10, 34, and 144. How could we represent each situation with an equation?

In the first situation, there is one shopping cart with length 34 and then an unknown number of carts with length 10. Similarly, Kiran has 34 dollars saved and then will save 10 each week for an unknown number of weeks. Both situations have one part of 34 and then equal parts of size 10 that all add together to 144. Their equation is $34 + 10x = 144$.

Since it takes 11 groups of 10 to get from 34 to 144, the value of $x$ in these two situations is $(144 - 34) \div 10$ or 11. There will be 11 shopping carts in each row, and it will take Kiran 11 weeks to raise the money for the guitar.

In the bakery situation, there is one part of 10 and then 144 equal parts of unknown size that all add together to 34. The equation is $10 + 144x = 34$. Since 24 is needed to get from 10 to 34, the value of $x$ is $(34 - 10) \div 144$ or $\frac{1}{6}$. There is $\frac{1}{6}$ cup of sugar in each giant muffin.

7.6.5 Reasoning about Equations and Tape Diagrams Date__________________

**Student Objective**: Let's use tape diagrams to write and solve equations with and without parentheses.

### 5.1: Algebra Talk: Seeing Structure

Solve each equation mentally.

$$x - 1 = 5$$

$$2(x - 1) = 10$$

$$3(x - 1) = 15$$

$$500 = 100(x - 1)$$

### 5.2: More Situations and Diagrams

Draw a tape diagram to represent each situation. Then write and solve an equation.

1. Each of 5 gift bags contains $x$ pencils. Tyler adds 3 more pencils to each bag. Altogether, the gift bags contain 20 pencils.

2. Noah drew an equilateral triangle with sides of length $x$ inches. He wants to decrease the length of each side by $5$ inches so the triangle is still equilateral and has a perimeter of 21 inches.

3. An art class charges each student \$2 to attend plus a fee for supplies. Today, \$20 was collected for the 4 students attending the class.

## Lesson 5 Summary

Equations with parentheses can represent a variety of situations.

1. Lin volunteers at a hospital and is preparing toy baskets for children who are patients. She adds 2 items to each basket, after which the supervisor's list shows that 140 toys have been packed into a group of 10 baskets. Lin wants to know how many toys were in each basket before she added the items.
2. A large store has the same number of workers on each of 2 teams to handle different shifts. They decide to add 10 workers to each team, bringing the total number of workers to 140. An executive at the company that runs this chain of stores wants to know how many employees were in each team before the increase.

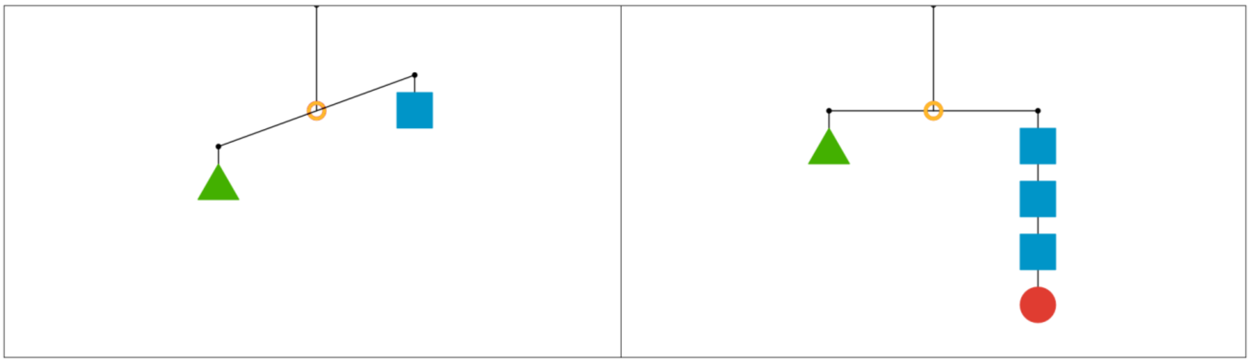
Each bag in the first story has an unknown number of toys, $x$, that is increased by 2. Then ten groups of $x + 2$ give a total of 140 toys. An equation representing this situation is $10(x + 2) = 140$. Since 10 times a number is 140, that number is 14, which is the total number of items in each bag. Before Lin added the 2 items there were $14 - 2$ or 12 toys in each bag.

The executive in the second story knows that the size of each team of $y$ employees has been increased by 10. There are now 2 teams of $y + 10$ each. An equation representing this situation is $2(y + 10) = 140$. Since 2 times an amount is 140, that amount is 70, which is the new size of each team. The value of $y$ is $70 - 10$ or 60. There were 60 employees on each team before the increase.

7.6.7 Reasoning about Solving Equations (Part 1)  Date_______________________

**Student Objective:** Let's see how a balanced hanger is like an equation and how moving its weights is like solving the equation.

## 7.1: Hanger Diagrams



In the two diagrams, all the triangles weigh the same and all the squares weigh the same.

For each diagram, come up with . . .

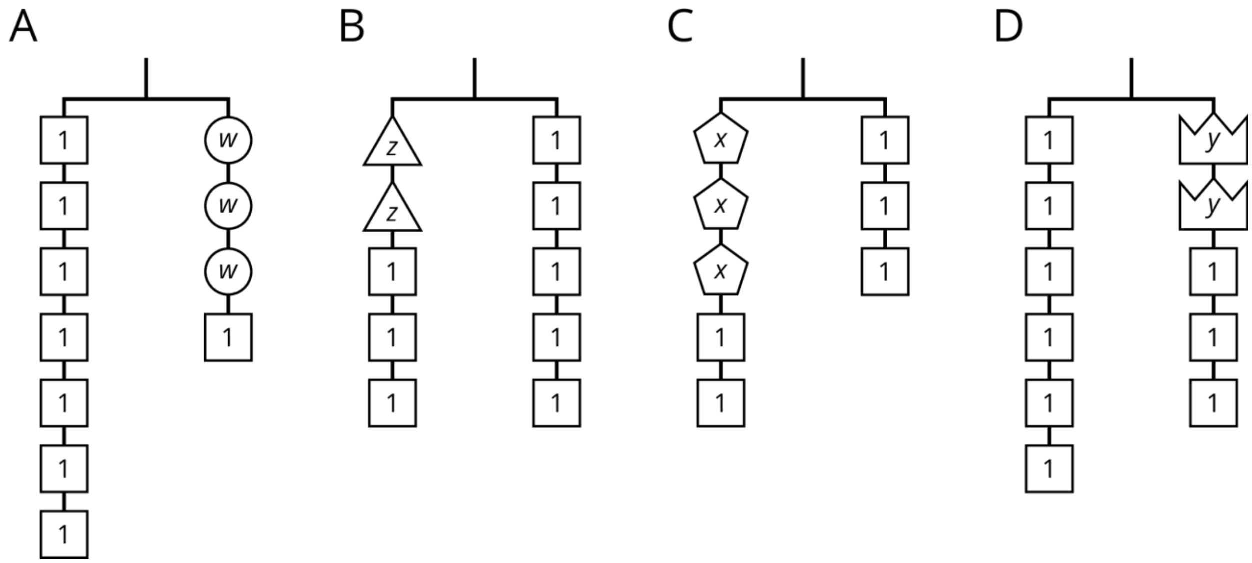
1. One thing that *must* be true

2. One thing that *could* be true

3. One thing that *cannot possibly* be true

## 7.2: Hanger and Equation Matching

On each balanced hanger, figures with the same letter have the same weight.




- $2\square + 3 = 5$
- $3\square + 2 = 3$
- $6 = 2\square + 3$
- $7 = 3\square + 1$

1. Match each hanger to an equation. Complete the equation by writing $x$, $y$, $z$, or $w$ in the empty box.
2. Find the solution to each equation. Use the hanger to explain what the solution means.

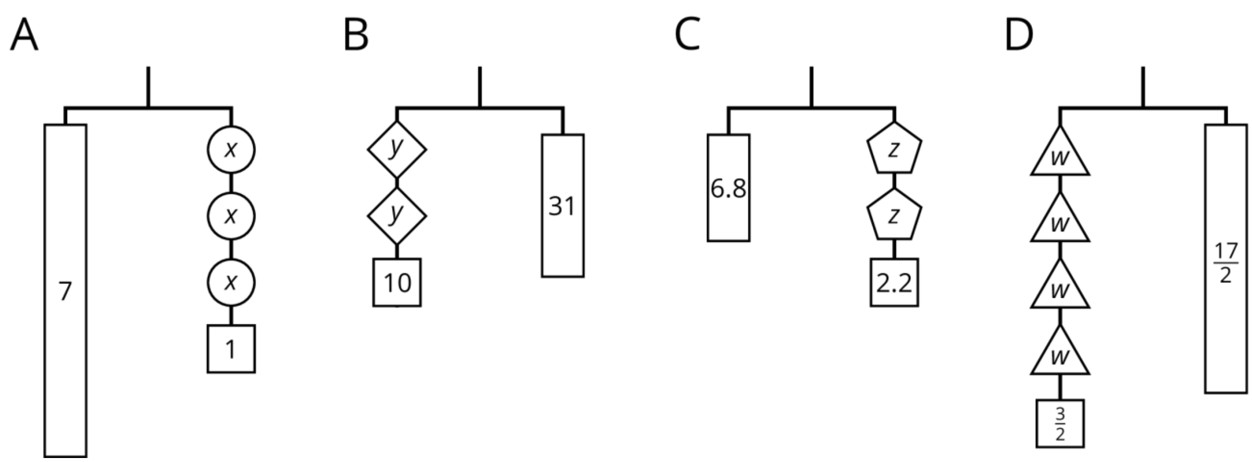
## 7.3: Use Hangers to Understand Equation Solving

Here are some balanced hangers where each piece is labeled with its weight. For each diagram:

1. Write an equation.

2. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the diagram.

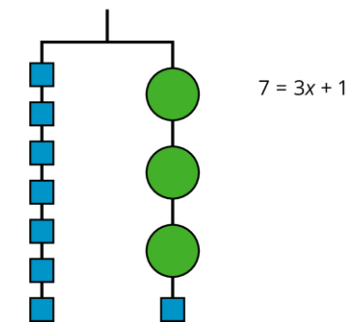
3. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the equation.



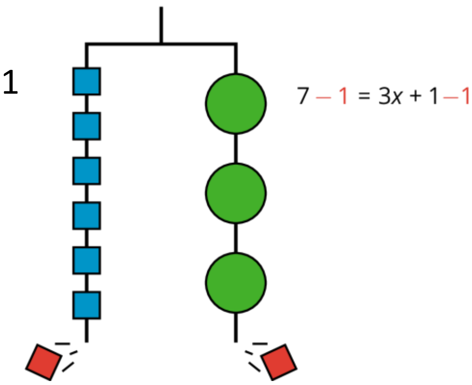
## Lesson 7 Summary

In this lesson, we worked with two ways to show that two amounts are equal: a balanced hanger and an equation. We can use a balanced hanger to think about steps to finding an unknown amount in an associated equation.

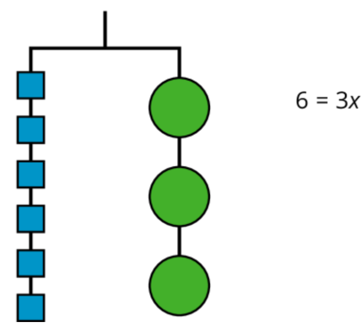
The hanger shows a total weight of 7 units on one side that is balanced with 3 equal, unknown weights and a 1-unit weight on the other. An equation that represents the relationship is $7 = 3x + 1$.



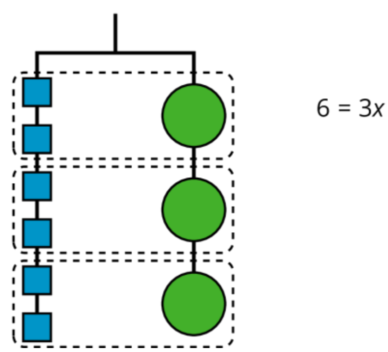

We can remove a weight of 1 unit from each side and the hanger will stay balanced. This is the same as subtracting 1 from each side of the equation.



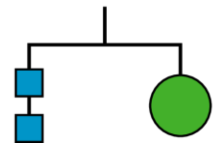

An equation for the new balanced hanger is $6 = 3x$.




So the hanger will balance with $\frac{1}{3}$ of the weight on each side: $\frac{1}{3} \cdot 6 = \frac{1}{3} \cdot 3x$.



The two sides of the hanger balance with these weights: 6 1-unit weights on one side and 3 weights of unknown size on the other side.



2 = $x$

Here is a concise way to write the steps above:

$$\begin{aligned} 7 &= 3x + 1 & & \\ 6 &= 3x & & \text{after subtracting 1 from each side} \\ 2 &= x & & \text{after multiplying each side by } \frac{1}{3} \end{aligned}$$

7.6.8 Reasoning about Solving Equations (Part 2) Date______________________

## Student Objective:

Let's use hangers to understand two different ways of solving equations with parentheses.

## 8.1: Equivalent to $2(x+3)$

Select **all** the expressions equivalent to $2(x+3)$.

a. $2 \cdot (x+3)$

b. $(x+3)2$

c. $2 \cdot x + 2 \cdot 3$

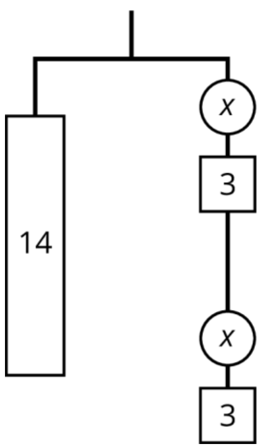
d. $2 \cdot x + 3$

e. $(2 \cdot x) + 3$

f. $(2+x)3$

## 8.2: Either Or

1. Explain why either of these equations could represent this hanger:

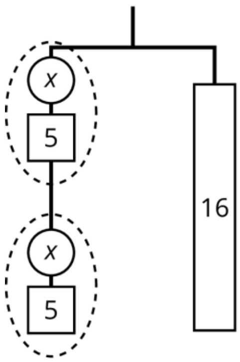



$14 = 2(x+3)$ or $14 = 2x + 6$

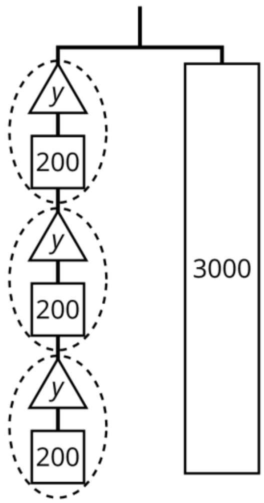
2. Find the weight of one circle. Be prepared to explain your reasoning.

## 8.3: Use Hangers to Understand Equation Solving, Again

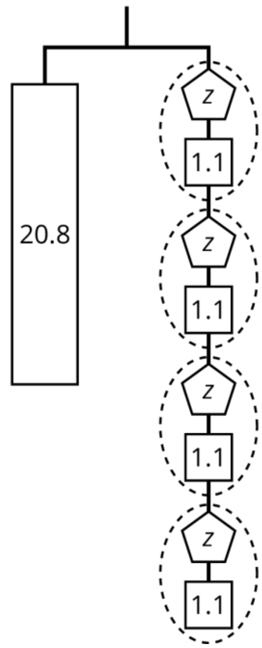
Here are some balanced hangers and equations. Each piece is labeled with its weight. Explain how to figure out the weight of a piece labeled with a letter by using properties of equality and of operations.



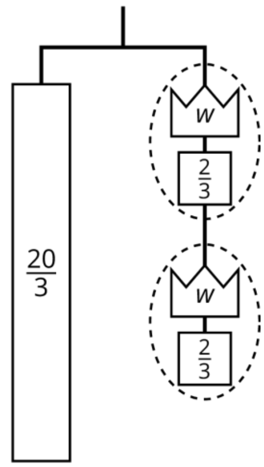

$$2(x+5)=16$$




$$3(y+200)=3{,}000$$



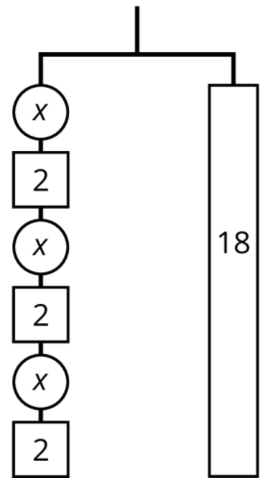

$$20.8=4(z+1.1)$$




$$\frac{20}{3}=2\left(w+\frac{2}{3}\right)$$

## Lesson 8 Summary

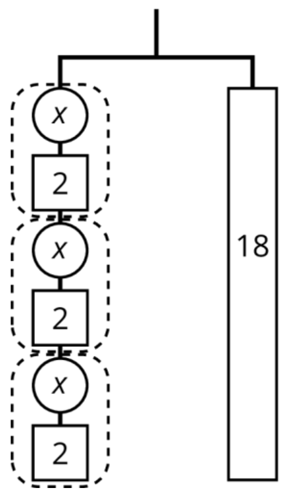
The balanced hanger shows 3 equal, unknown weights and 3 2-unit weights on the left and an 18-unit weight on the right.




There are 3 unknown weights plus 6 units of weight on the left. We could represent this balanced hanger with an equation and solve the equation the same way we did before.

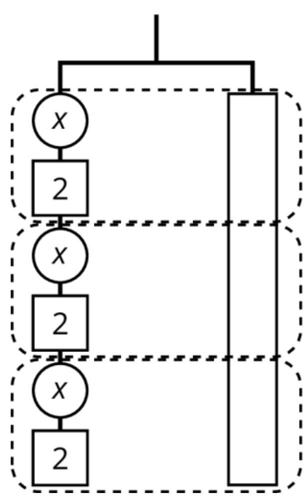
$$\begin{aligned} 3x + 6 &= 18 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

Since there are 3 groups of $x + 2$ on the left, we could represent this hanger with a different equation: $3(x + 2) = 18$.



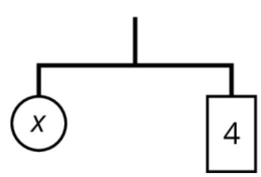

$3(x + 2) = 18$

The two sides of the hanger balance with these weights: 3 groups of $x + 2$ on one side, and 18, or 3 groups of 6, on the other side.



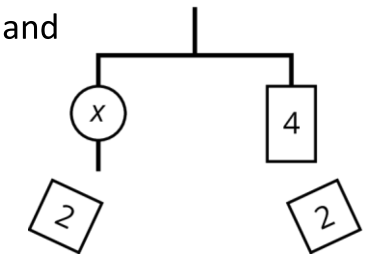

$3(x + 2) = 18$

The two sides of the hanger will balance with $\frac{1}{3}$ of the weight on each side: $\frac{1}{3} \cdot 3(x + 2) = \frac{1}{3} \cdot 18$.



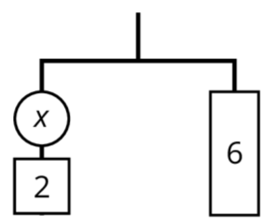

$x = 4$

We can remove 2 units of weight from each side, and the hanger will stay balanced. This is the same as subtracting 2 from each side of the equation.




$x + 2 = 4 + 2$

An equation for the new balanced hanger is $x = 4$. This gives the solution to the original equation.




$x + 2 = 6$

Here is a concise way to write the steps above:

$$
\begin{aligned}
3(x+2) &= 18 \\
x + 2 &= 6 \qquad \text{after multiplying each side by } \frac{1}{3} \\
x &= 4 \qquad \text{after subtracting 2 from each side}
\end{aligned}
$$

7.6.10 Different Options for Solving One Equation Date______________________

## Student Objective:

Let’s think about which way is easier when we solve equations with parentheses.

## 10.1: Algebra Talk: Solve Each Equation

$$100(x-3)=1{,}000$$

$$500(x-3)\ =\ 5{,}000$$

$$0.03(x-3)=0.3$$

$$0.72(x+2)=7.2$$

$$\frac{1}{7}(x+2)=\frac{10}{7}$$

## 10.2: Analyzing Solution Methods

Four students each attempted to solve the equation $2(x-9)=10$, but got different solutions. Here are their methods. Do you agree with any of their methods, and why? Hint: try checking their solutions.

Noah’s method:

$$\begin{aligned}
2(x-9) &= 10 \\
2(x-9)+9 &= 10+9 && \text{add 9 to each side} \\
2x &= 19 \\
2x \div 2 &= 19 \div 2 && \text{divide each side by 2} \\
x &= \frac{19}{2}
\end{aligned}$$

Elena’s method:

$$\begin{aligned}
2(x-9) &= 10 \\
2x-18 &= 10 && \text{apply the distributive property} \\
2x-18-18 &= 10-18 && \text{subtract 18 from each side} \\
2x &= -8 \\
2x \div 2 &= -8 \div 2 && \text{divide each side by 2} \\
x &= -4
\end{aligned}$$

Maya's method:

$$\begin{aligned}
2(x-9) &= 10 \\
2x-18 &= 10 && \text{apply the distributive property} \\
2x-18+18 &= 10+18 && \text{add 18 to each side} \\
2x &= 28 \\
2x \div 2 &= 28 \div 2 && \text{divide each side by 2} \\
x &= 14
\end{aligned}$$

Connor's method:

$$\begin{aligned}
2(x-9) &= 10 \\
x-9 &= 5 && \text{divide each side by 2} \\
x-9+9 &= 5+9 && \text{add 9 to each side} \\
x &= 14
\end{aligned}$$

## 10.3: Solution Pathways

For each equation, try to solve the equation using each method (dividing each side first, or applying the distributive property first). Some equations are easier to solve by one method than the other. When that is the case, stop doing the harder method and write down the reason you stopped.

1. $2{,}000(x - 0.03) = 6{,}000$

2. $2(x + 1.25) = 3.5$

3. $\frac{1}{4}(4 + x) = \frac{4}{3}$

4. $-10(x - 1.7) = -3$

5. $5.4 = 0.3(x + 8)$

## Lesson 10 Summary

Equations can be solved in many ways. In this lesson, we focused on equations with a specific structure, and two specific ways to solve them.

Suppose we are trying to solve the equation $\frac{4}{5}(x+27)=16$. Two useful approaches are:

- divide each side by $\frac{4}{5}$
- apply the distributive property

In order to decide which approach is better, we can look at the numbers and think about which would be easier to compute. We notice that $\frac{4}{5}\cdot 27$ will be hard, because 27 isn't divisible by 5. But $16 \div \frac{4}{5}$ gives us $16\cdot\frac{5}{4}$, and 16 is divisible by 4. Dividing each side by $\frac{4}{5}$ gives:

$$\begin{aligned} \frac{4}{5}(x+27) &= 16 \\ \frac{5}{4}\cdot\frac{4}{5}(x+27) &= 16\cdot\frac{5}{4} \\ x+27 &= 20 \\ x &= -7 \end{aligned}$$

Sometimes the calculations are simpler if we first use the distributive property. Let's look at the equation $100(x+0.06)=21$. If we first divide each side by 100, we get $\frac{21}{100}$ or 0.21 on the right side of the equation. But if we use the distributive property first, we get an equation that only contains whole numbers.

$$\begin{aligned} 100(x+0.06) &= 21 \\ 100x+6 &= 21 \\ 100x &= 15 \\ x &= \frac{15}{100} \end{aligned}$$

7.6.11 Using Equations to Solve Problems Date______________________

**Student Objective**: Let's use tape diagrams, equations, and reasoning to solve problems.

## 11.1 20% Off

An item costs $x$ dollars and then a 20% discount is applied. Select **all** the expressions that could represent the price of the item after the discount.

a. $\frac{20}{100}x$

b. $x - \frac{20}{100}x$

c. $(1 - 0.20)x$

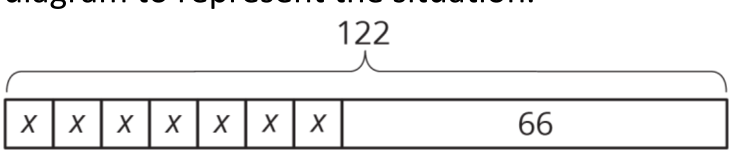
d. $\frac{100-20}{100}x$

e. $0.80x$

f. $(100 - 20)x$

## 11.2: At the Fair

1. Tyler is making invitations to the fair. He has already made some of the invitations, and he wants to finish the rest of them within a week. He is trying to spread out the remaining work, to make the same number of invitations each day. Tyler draws a diagram to represent the situation.




A. Explain how each part of the situation is represented in Tyler's diagram:

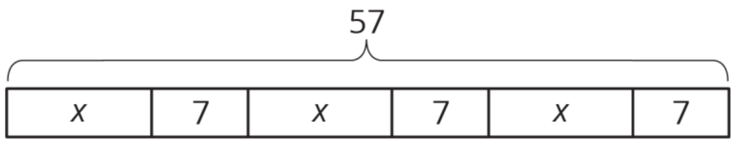
How many total invitations Tyler is trying to make.____________________

How many invitations he has made already.____________________

How many days he has to finish the invitations.________________

B. Use Tyler's diagram to write and solve an equation that represents the situation.

2. Noah and his sister are making prize bags for a game at the fair. Noah is putting 7 pencil erasers in each bag. His sister is putting in some number of stickers. After filling 3 of the bags, they have used a total of 57 items.




A. Explain how the diagram represents the situation.

B. Noah writes the equation $3(x+7)=57$ to represent the situation. Do you agree with him? Explain your reasoning.

C. Solve the equation to determine how many stickers is Noah's sister putting in each prize bag. Check your solution.

**11.3:** Write and solve an equation to answer the question. Check your solution. If you get stuck, consider drawing a diagram.

1. A store is having a sale where all shoes are discounted by 20%. Diego has a coupon for $\$3$ off of the regular price for one pair of shoes. The store first applies the coupon and then takes 20% off of the reduced price. If Diego pays $\$18.40$ for a pair of shoes, what was their original price before the sale and without the coupon?

2. Before the sale, the store had 100 pairs of flip flops in stock. After selling some, they notice that $\frac{3}{5}$ of the flip flops they have left are blue. If the store has 39 pairs of blue flip flops, how many pairs of flip flops (any color) have they sold?

3. On the morning of the sale, the store donated 50 pairs of shoes to a homeless shelter. Then they sold 64% of their remaining inventory during the sale. If the store had 288 pairs after the donation and the sale, how many pairs of shoes did they have at the start?

## Lesson 11 Summary

Many problems can be solved by writing and solving an equation. Here is an example:

Clare ran 4 miles on Monday. Then for the next six days, she ran the same distance each day. She ran a total of 22 miles during the week. How many miles did she run on each of the 6 days?

One way to solve the problem is to represent the situation with an equation, $4 + 6x = 22$, where $x$ represents the distance, in miles, she ran on each of the 6 days. Solving the equation gives the solution to this problem.

$$\begin{aligned} 4 + 6x &= 22 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

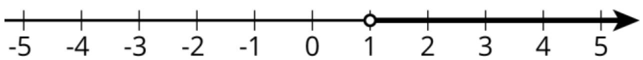
7.6.13 Reintroducing Inequalities Date______________________

## Student Objective:

Let’s work with inequalities.

## 13.1: Greater Than One

The number line shows values of $x$ that make the inequality $x > 1$ true.




1. Select **all** the values of $x$ from this list that make the inequality $x > 1$ true.

   a. 3

   b. -3

   c. 1

   d. 700

   e. 1.05

2. Name two more values of $x$ that are solutions to the inequality.

## 13.2: The Roller Coaster

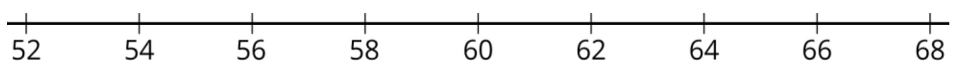
A sign next to a roller coaster at an amusement park says, “You must be at least 60 inches tall to ride.” Noah is happy to know that he is tall enough to ride.

1. Noah is $x$ inches tall. Which of the following can be true: $x > 60$, $x = 60$, or $x < 60$? Explain how you know.

2. Noah’s friend is 2 inches shorter than Noah. Can you tell if Noah’s friend is tall enough to go on the ride? Explain or show your reasoning.

3. List one possible height for Noah that means that his friend is tall enough to go on the ride, and another that means that his friend is too short for the ride.

4. On the number line below, show all the possible heights that Noah’s friend could be.




5. Noah's friend is $y$ inches tall. Use $y$ and any of the symbols $<, =, >$ to express this height.

## 13.3: Is the Inequality True or False?

The table shows four inequalities and four possible values for $x$. Decide whether each value makes each inequality true, and complete the table with "true" or "false." Discuss your thinking with your partner. If you disagree, work to reach an agreement.

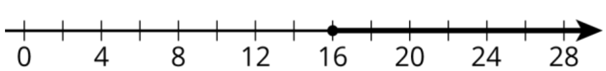
| $x$ | 0 | 100 | -100 | 25 |
|---|---|---|---|---|
| $x \leq 25$ | | | | |
| $100 < 4x$ | | | | |
| $-3x > -75$ | | | | |
| $10 \geq 35 - x$ | | | | |

### *Are you ready for more?*

Find an example of in inequality used in the real world and describe it using a number line.

## Lesson 13 Summary

We use inequalities to describe a range of numbers. In many places, you are allowed to get a driver's license when you are at least 16 years old. When checking if someone is old enough to get a license, we want to know if their age is at least 16. If $h$ is the age of a person, then we can check if they are allowed to get a driver's license by checking if their age makes the inequality $h > 16$ (they are older than 16) or the equation $h = 16$ (they are 16) true. The symbol $\geq$, pronounced "greater than or equal to," combines these two cases and we can just check if $h \geq 16$ (their age is greater than or equal to 16). The inequality $h \geq 16$ can be represented on a number line:



7.6.14 Finding Solutions to Inequalities in Context Date______________________

**Student Objective:** Let's solve more complicated inequalities.

## 14.1: Solutions to Equations and Solutions to Inequalities

1. Solve $-x = 10$
2. Find 2 solutions to $-x > 10$
3. Solve $2x = -20$
4. Find 2 solutions to $2x > -20$

## 14.2: Earning Money for Soccer Stuff

1. Andre has a summer job selling magazine subscriptions. Each week, he earns $25 plus $3 for every subscription he sells. Andre hopes to make at least enough money this week to buy a new pair of soccer cleats. Let $n$ represent the number of magazine subscriptions Andre sells this week. An expression for the amount of money he makes this week is $25 + 3n$.

A. The least expensive pair of cleats Andre wants costs $68. Write and solve an equation to find out how many magazine subscriptions Andre needs to sell to buy the cleats.

B. If Andre sold 16 magazine subscriptions this week, would he reach his goal?________

C. What are some other numbers of magazine subscriptions Andre could have sold and still reached his goal?

D. Write an *inequality* expressing that Andre wants to make at least $68.

E. Write an inequality to describe the number of subscriptions Andre must sell to reach his goal.

## 14.3: Granola Bars and Savings

1. Kiran has $100 saved in a bank account. (The account doesn't earn interest.) He asked Clare to help him figure out how much he could take out each month if he needs to have at least $25 in the account a year from now.

A. Clare wrote the inequality $-12x + 100 \geq 25$, where $x$ represents the amount Kiran takes out each month. What does $-12x$ represent?

B. Find some values of $x$ that would work for Kiran.

C. We could express *all* the values that would work using either $x \leq 6.25 \text{ or } x \geq 6.25$. Which one should we use?

2. Diego has budgeted $35 from his summer job earnings to buy shorts and socks for soccer. He needs 5 pairs of socks and a pair of shorts. The socks cost different amounts in different stores. The shorts he wants cost $19.95.

   A. Let $x$ represent the price of one pair of socks. Write an expression for the total cost of the socks and shorts.

   B. Write and solve an equation that says that Diego spent exactly $35 on the socks and shorts.

   C. Write an inequality to represent the amount Diego can spend on a single pair of socks.

3. A teacher wants to buy 9 boxes of granola bars for a school trip. Each box usually costs \$7, but many grocery stores are having a sale on granola bars this week. Different stores are selling boxes of granola bars at different discounts.

   A. If $x$ represents the dollar amount of the discount, then the amount the teacher will pay can be expressed as $9(7-x)$. In this expression, what does the quantity $7-x$ represent?

   B. The teacher has \$36 to spend on the granola bars. The equation $9(7-x)=36$ represents a situation where she spends all \$36. Solve this equation.

   C. What does the solution mean in this situation?

   D. The teacher does not have to spend all \$36. Write an inequality relating 36 and $9(7-x)$ representing this situation.

   E. The solution to this inequality must either look like $x \geq 3$ or $x \leq 3$. Which do you think it is? Explain your reasoning.

## Lesson 14 Summary

Suppose Elena has \$5 and sells pens for \$1.50 each. Her goal is to save \$20. We could solve the equation $1.5x + 5 = 20$ to find the number of pens, $x$, that Elena needs to sell in order to save *exactly* \$20. Adding -5 to both sides of the equation gives us $1.5x = 15$, and then dividing both sides by $1.5$ gives the solution $x = 10$ pens.

What if Elena wants to have some money left over? The inequality $1.5x + 5 > 20$ tells us that the amount of money Elena makes needs to be *greater* than \$20. The solution to the previous equation will help us understand what the solutions to the inequality will be. We know that if she sells 10 pens, she will make \$20. Since each pen gives her more money, she needs to sell *more* than 10 pens to make more than \$20. So the solution to the inequality is $x > 10$

7.6.15 Efficiently Solving Inequalities Date______________________

**Student Objective:** Let's solve more complicated inequalities.

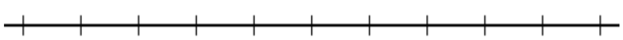
## 15.1: Lots of Negatives

Here is an inequality: $-x \geq -4$.

1. Predict what you think the solutions on the number line will look like.

2. Select **all** the values that are solutions to $-x \geq -4$:
   - a. 3
   - b. -3
   - c. 4
   - d. -4
   - e. 4.001
   - f. -4.001

3. Graph the solutions to the inequality on the number line:



## 15.2: Inequalities with Tables

1. Let's investigate the inequality $x - 3 > -2$.

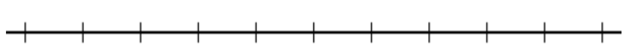
| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $x-3$ | -7 | | -5 | | | | -1 | | 1 |

   A. Complete the table.

   B. For which values of $x$ is it true that $x - 3 = -2$?

   C. For which values of $x$ is it true that $x - 3 > -2$?

   D. Graph the solutions to $x - 3 > -2$ on the number line:



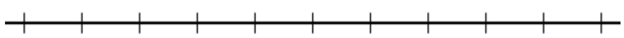
2. Here is an inequality: $2x < 6$.

   A. Predict which values of $x$ will make the inequality $2x < 6$ true.

   B. Complete the table. Does it match your prediction?

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $2x$ | | | | | | | | | |

   C. Graph the solutions to $2x < 6$ on the number line:



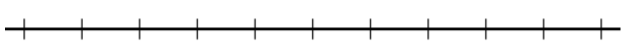
3. Here is an inequality: $-2x < 6$.

   A. Predict which values of $x$ will make the inequality $-2x < 6$ true.

   B. Complete the table. Does it match your prediction?

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $-2x$ | | | | | | | | | |

   C. Graph the solutions to $-2x < 6$ on the number line:



   D. How are the solutions to $2x < 6$ different from the solutions to $-2x < 6$?

### 15.3: Which Side are the Solutions?

1. Let's investigate $-4x + 5 \geq 25$.
   A. Solve $-4x + 5 = 25$.

   B. Is $-4x + 5 \geq 25$ true when $x$ is 0? What about when $x$ is 7? What about when $x$ is -7?

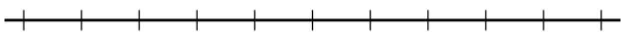
   C. Graph the solutions to $-4x + 5 \geq 25$ on the number line.

2. Let's investigate $\frac{4}{3}x + 3 < \frac{23}{3}$.
   A. Solve $\frac{4}{3}x + 3 = \frac{23}{3}$.

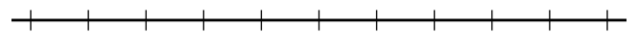
   B. Is $\frac{4}{3}x + 3 < \frac{23}{3}$ true when $x$ is 0?

   C. Graph the solutions to $\frac{4}{3}x + 3 < \frac{23}{3}$ on the number line.

3. Solve $3(x+4) > 17.4$ and graph the solutions on the number line.



4. Solve $-3\left(x-\frac{4}{3}\right) \leq 6$ and graph the solutions on the number line.



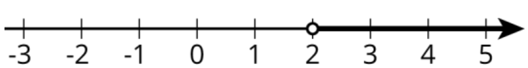
## Lesson 15 Summary

Here is an inequality: $3(10-2x) < 18$. The solution to this inequality is all the values you could use in place of $x$ to make the inequality true.

In order to solve this, we can first solve the related equation $3(10-2x) = 18$ to get the solution $x = 2$. That means 2 is the boundary between values of $x$ that make the inequality true and values that make the inequality false.

To solve the inequality, we can check numbers greater than 2 and less than 2 and see which ones make the inequality true.

Let’s check a number that is greater than 2: $x = 5$. Replacing $x$ with 5 in the inequality, we get $3(10-2\cdot 5) < 18$ or just $0 < 18$. This is true, so $x = 5$ is a solution. This means that all values greater than 2 make the inequality true. We can write the solutions as $x > 2$ and also represent the solutions on a number line:




Notice that 2 itself is not a solution because it's the value of $x$ that makes $3(10-2x)$ *equal* to 18, and so it does not make $3(10-2x) < 18$ true.

For confirmation that we found the correct solution, we can also test a value that is less than 2. If we test $x = 0$, we get $3(10-2\cdot 0) < 18$ or just $30 < 18$. This is false, so $x = 0$ and all values of $x$ that are less than 2 are not solutions.

7.6.16 Interpreting Inequalities Date______________________

**Student Objective:** Let's write inequalities.

## 16.1: Solve Some Inequalities!

For each inequality, find the value or values of $x$ that make it true.

1. $8x + 21 \leq 56$
2. $56 < 7(7 - x)$

## 16.2: Club Activities Matching

Choose the inequality that best matches each given situation. Explain your reasoning.

1. The Garden Club is planting fruit trees in their school's garden. There is one large tree that needs 5 pounds of fertilizer. The rest are newly planted trees that need $\frac{1}{2}$ pound fertilizer each.
   a. $25x + 5 \leq \frac{1}{2}$
   b. $\frac{1}{2}x + 5 \leq 25$
   c. $\frac{1}{2}x + 25 \leq 5$
   d. $5x + \frac{1}{2} \leq 25$

2. The Chemistry Club is experimenting with different mixtures of water with a certain chemical (sodium polyacrylate) to make fake snow.
   To make each mixture, the students start with some amount of water, and then add $\frac{1}{7}$ of that amount of the chemical, and then 9 more grams of the chemical. The chemical is expensive, so there can't be more than a certain number of grams of the chemical in any one mixture.
   a. $\frac{1}{7}x + 9 \leq 26.25$
   b. $9x + \frac{1}{7} \leq 26.25$
   c. $26.25x + 9 \leq \frac{1}{7}$
   d. $\frac{1}{7}x + 26.25 \leq 9$

3. The Hiking Club is on a hike down a cliff. They begin at an elevation of 12 feet and descend at the rate of 3 feet per minute.

   a. $37x - 3 \geq 12$

   b. $3x - 37 \geq 12$

   c. $12 - 3x \geq -37$

   d. $12x - 37 \geq -3$

4. The Science Club is researching boiling points. They learn that at high altitudes, water boils at lower temperatures. At sea level, water boils at $212°\mathrm{F}$. With each increase of 500 feet in elevation, the boiling point of water is lowered by about $1°\mathrm{F}$.

   a. $212 - \frac{1}{500}e < 195$

   b. $\frac{1}{500}e - 195 < 212$

   c. $195 - 212e < \frac{1}{500}$

   d. $212 - 195e < \frac{1}{500}$

## Lesson 16 Summary

We can represent and solve many real-world problems with inequalities. Writing the inequalities is very similar to writing equations to represent a situation. The expressions that make up the inequalities are the same as the ones we have seen in earlier lessons for equations. For inequalities, we also have to think about how expressions compare to each other, which one is bigger, and which one is smaller. Can they also be equal?

For example, a school fundraiser has a minimum target of $500. Faculty have donated $100 and there are 12 student clubs that are participating with different activities. How much money should each club raise to meet the fundraising goal? If $n$ is the amount of money that each club raises, then the solution to $100 + 12n = 500$ is the minimum amount each club has to raise to meet the goal. It is more realistic, though, to use the inequality $100 + 12n \geq 500$ since the more money we raise, the more successful the fundraiser will be. There are many solutions because there are many different amounts of money the clubs could raise that would get us above our minimum goal of $500.

7.6.17 Modeling with Inequalities Date____________________

**Student Objective:** Let's look at solutions to inequalities.

## 17.1: Possible Values

The stage manager of the school musical is trying to figure out how many sandwiches he can order with the $83 he collected from the cast and crew. Sandwiches cost $5.99 each, so he lets $x$ represent the number of sandwiches he will order and writes $5.99x \leq 83$. He solves this to 2 decimal places, getting $x \leq 13.86$.

Which of these are valid statements about this situation? (Select **all** that apply.)

a. He can call the sandwich shop and order exactly 13.86 sandwiches.
b. He can round up and order 14 sandwiches.
c. He can order 12 sandwiches.
d. He can order 9.5 sandwiches.
e. He can order 2 sandwiches.
f. He can order -4 sandwiches.

## 17.2: Elevator

A mover is loading an elevator with many identical 48-pound boxes.
The mover weighs 185 pounds. The elevator can carry at most 2000 pounds.

1. Write an inequality that says that the mover will not overload the elevator on a particular ride. Check your inequality with your partner.
2. Solve your inequality.

3. Graph the solution to your inequality on a number line.

4. If the mover asked, “How many boxes can I load on this elevator at a time?” what would you tell them?

## 17.3: Giving Advice

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information you need.
3. Explain to your partner how you are using the information to solve the problem.
4. Solve the problem and explain your reasoning to your partner.

If your teacher gives you the *data card*:

1. Silently read the information on your card.
2. Ask your partner "What specific information do you need?" and wait for your partner to *ask* for information. *Only* give information that is on your card. (Do not figure out anything for your partner!)
3. After your partner solves the problem, ask them to explain their reasoning and listen to their explanation.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

## Lesson 17 Summary

We can represent and solve many real-world problems with inequalities. Whenever we write an inequality, it is important to decide what quantity we are representing with a variable. After we make that decision, we can connect the quantities in the situation to write an expression, and finally, the whole inequality.

As we are solving the inequality or equation to answer a question, it is important to keep the meaning of each quantity in mind. This helps us to decide if the final answer makes sense in the context of the situation.

For example: Han has 50 centimeters of wire and wants to make a square picture frame with a loop to hang it that uses 3 centimeters for the loop. This situation can be represented by $3 + 4s = 50$, where $s$ is the length of each side (if we want to use all the wire). We can also use $3 + 4s \leq 50$ if we want to allow for solutions that don't use all the wire. In this case, any positive number that is less or equal to 11.75 cm is a solution to the inequality. Each solution represents a possible side length for the picture frame since Han can bend the wire at any point. In other situations, the variable may represent a quantity that increases by whole numbers, such as with numbers of magazines, loads of laundry, or students. In those cases, only whole-number solutions make sense.