Math 7 Period_______ Name____________________

7.3 Measuring Circles Date___________________

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OVERVIEW

In this unit, students learn to understand and use the term "circle" to mean the set of points that are equally distant from a point called the "center." They gain an understanding of why the circumference of a circle is proportional to its diameter, with constant of proportionality π. They see informal derivations of the fact that the area of a circle is equal to π times the square of its radius. Students use the relationships of circumference, radius, diameter, and area of a circle to find lengths and areas, expressing these in terms of π or using appropriate approximations of π to express them numerically.

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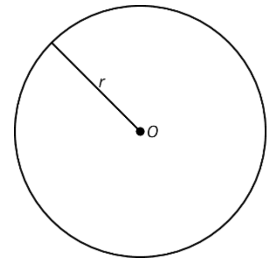
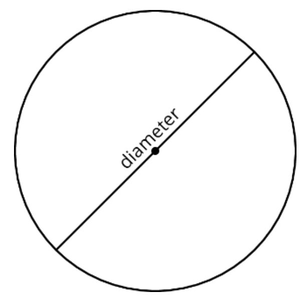
New or Recently Introduced Terms

- **area of a circle -** The area of a circle whose radius is r units is πr^2 square units.



- **Circle -** A circle of radius r with center O is the set of all points that are a distance r units from O.
  To draw a circle of radius 3 and center O, use a compass to draw all the points at a distance 3 from O.
- **Circumference -** The circumference of a circle is the distance around the circle. If you imagine the circle as a piece of string, it is the length of the string. If the circle has radius r then the circumference is $2\pi r$.
- **Diameter -** A line segment that has endpoints on a circle and passes through the center is called a diameter of the circle. The length of this segment is also called the diameter.



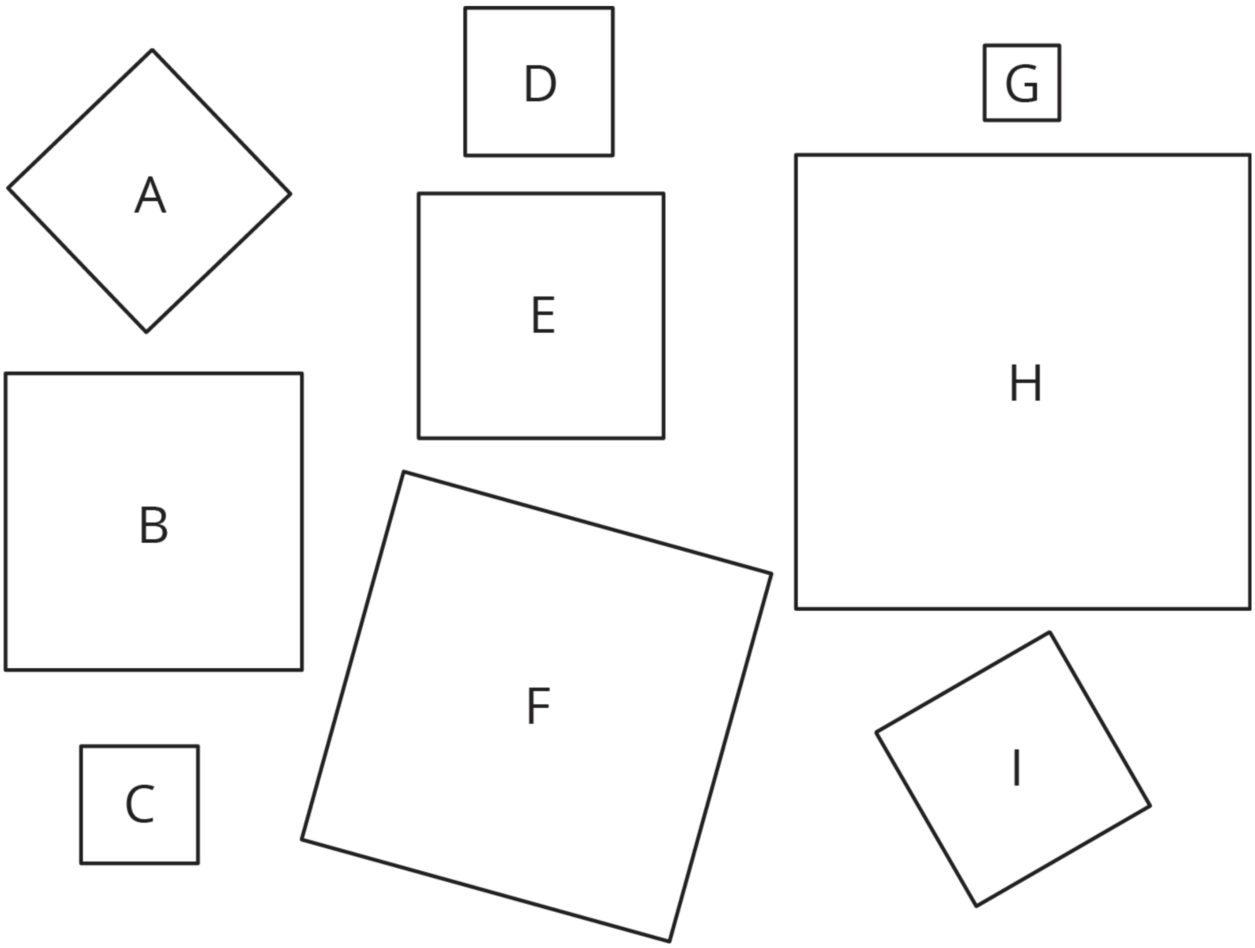
- **pi (π) -** The Greek letter π (pronounced "pie") stands for the number that is the constant of proportionality between the circumference of a circle and its diameter. If d is the diameter and C is the circumference, then $C=\pi d$.
- **Radius -** The distance from the center of a circle to any point on the circle. Also the corresponding line segment from the center to a point on the circle.

7.3.1 How Well Can You Measure Date______________________

Student Objective: See how accurately we can measure.

1.2: Perimeter of a Square

Here are nine squares.



Your teacher will assign your group three of these squares to examine more closely.

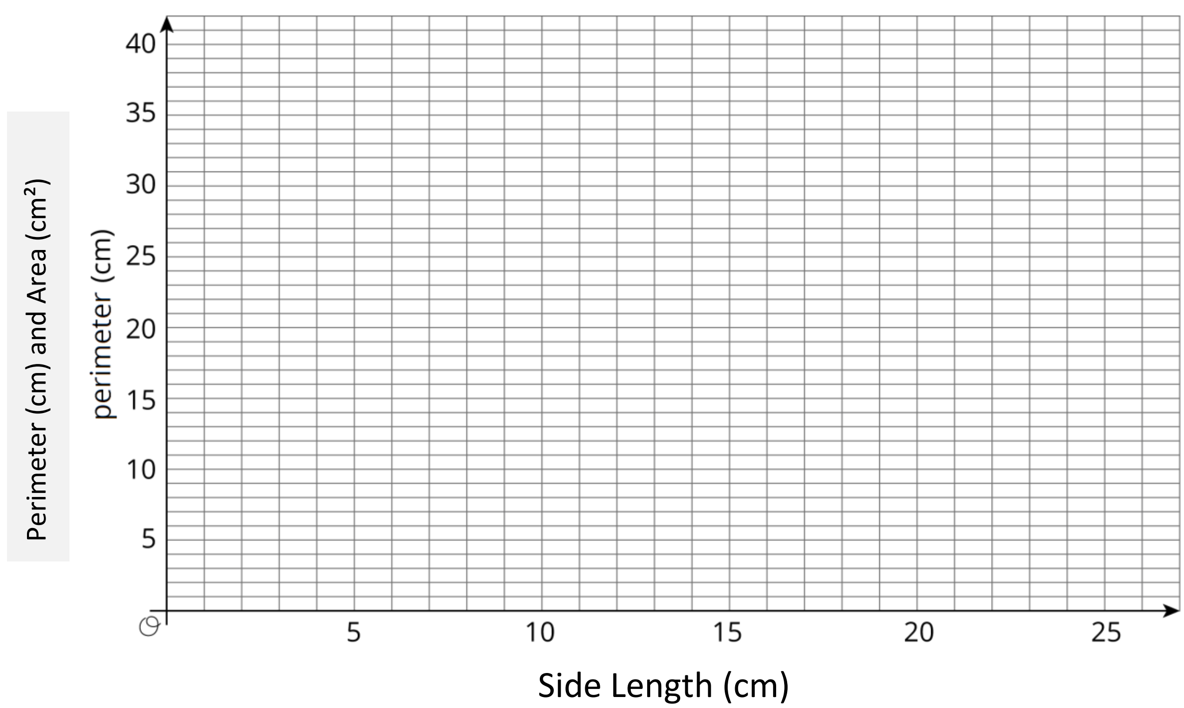
1. For each of your assigned squares, measure the side length of the square in centimeters. Record your measurements and calculations in the table. Did you get the same measurements as other students in the room? Why do you think this is?

	Side Length (cm)	Perimeter (cm) P =
Square A		
Square B		
Square C		
Square D		
Square E		
Square F		
Square G		
Square H		
Square I		

	Side Length (cm)	Area (cm^2) A =
Square A		
Square B		
Square C		
Square D		
Square E		
Square F		
Square G		
Square H		
Square I		

2. Plot the values from the tables on the coordinate plane. Use different colors for the two tables.

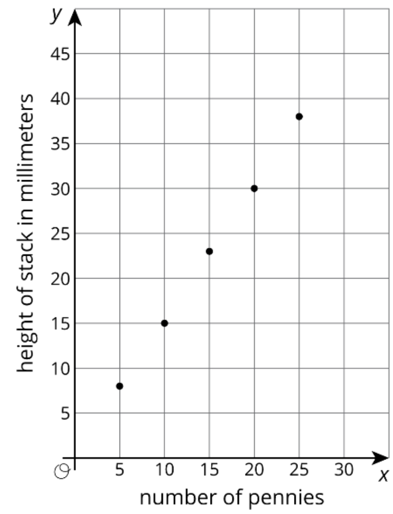
Comparing Measurements



3. How is the relationship between the side length and area of a square the same as the relationship between the side length and perimeter of a square from the previous activity? How is it different?

Lesson 1 Summary

When we measure the values for two related quantities, plotting the measurements in the coordinate plane can help us decide if it makes sense to model them with a proportional relationship. If the points are close to a line through $(0,0)$, then a proportional relationship is a good model. For example, here is a graph of the values for the height, measured in millimeters, of different numbers of pennies placed in a stack.



Because the points are close to a line through $(0,0)$, the height of the stack of pennies appears to be proportional to the number of pennies in a stack. This makes sense because we can see that the heights of the pennies only vary a little bit.

number of pennies	grams	grams per penny
1	3.1	3.1
2	5.6	2.8
5	13.1	2.6
10	25.6	2.6

An additional way to investigate whether or not a relationship is proportional is by making a table. Here is some data for the weight of different numbers of pennies in grams, along with the corresponding number of grams per penny.

Though we might expect this relationship to be proportional, the quotients are not very close to one another. In fact, the metal in pennies changed in 1982, and older pennies are heavier. This explains why the weight per penny for different numbers of pennies are so different!

7.3.2 Exploring Circles Date______________________

Student Objective: Explore circles.

2.1: How Do You Figure?

Here are two figures.

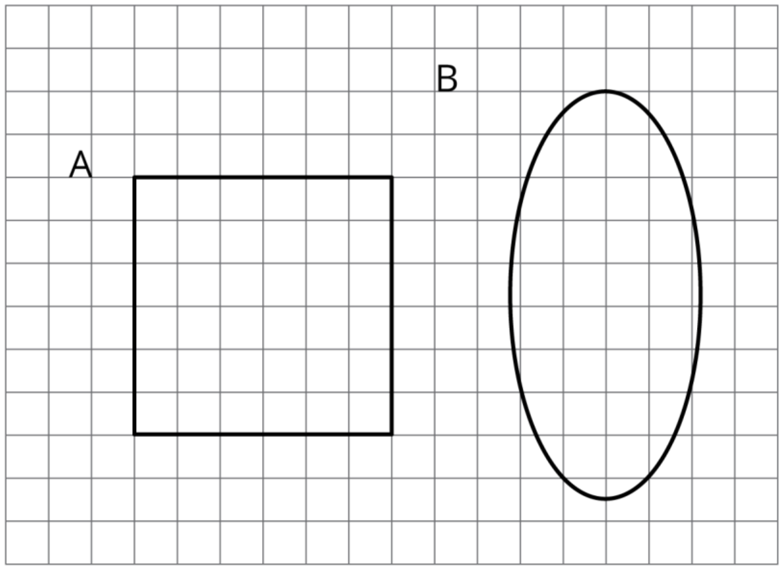


Figure C looks more like Figure A than like Figure B. Sketch what Figure C might look like. Explain your reasoning.

2.2: Sorting Round Objects

Your teacher will give you some pictures of different objects.

1. How could you sort these pictures into two groups? Be prepared to share your reasoning.

2. Work with your partner to sort the pictures into the categories that your class has agreed on. Pause here so your teacher can review your work.
3. What are some characteristics that all **circles** have in common?

4. Put the circular objects in order from smallest to largest.

5. Select one of the pictures of a circular object. What are some ways you could measure the actual size of your circle?

2.4: Drawing Circles

Draw and label each circle.

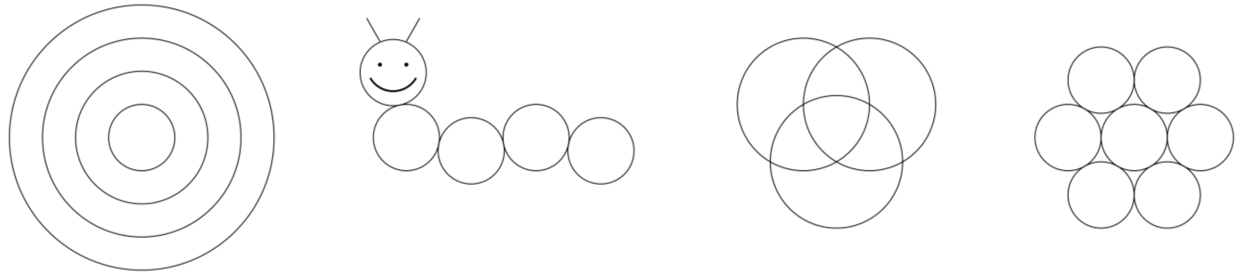
1. Circle A, with a **diameter** of 6 cm.

2. Circle B, with a **radius** of 5 cm. Pause here so your teacher can review your work.

3. Circle C, with a radius that is equal to Circle A's diameter.

4. Circle D, with a diameter that is equal to Circle B's radius.

5. Use a compass to recreate one of these designs.

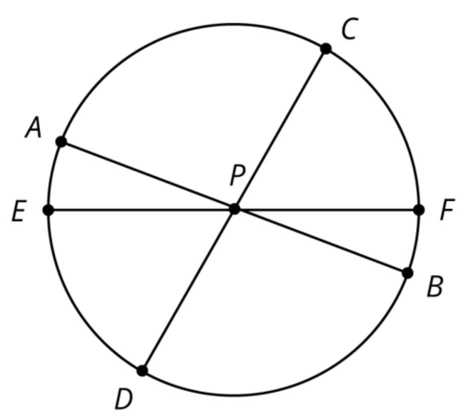


Lesson 2 Summary

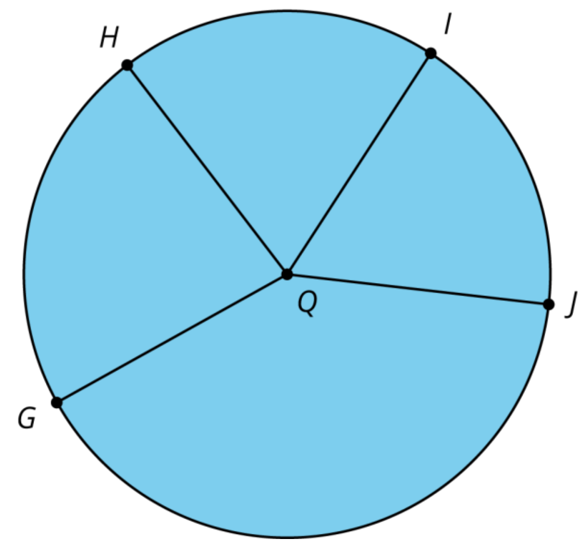
A **circle** consists of all of the points that are the same distance away from a particular point called the *center* of the circle.

A segment that connects the center with any point on the circle is called a **radius**. For example, segments QG, QH, QI, and QJ are all radii of circle 2. (We say one radius and two radii.) The length of any radius is always the same for a given circle. For this reason, people also refer to this distance as the *radius* of the circle.

circle 1



circle 2



A segment that connects two opposite points on a circle (passing through the circle's center) is called a **diameter**. For example, segments AB, CD, and EF are all diameters of circle 1. All diameters in a given circle have the same length because they are composed of two radii. For this reason, people also refer to the length of such a segment as the *diameter* of the circle.

The **circumference** of a circle is the distance around it. If a circle was made of a piece of string and we cut it and straightened it out, the circumference would be the length of that string. A circle always encloses a circular region. The region enclosed by circle 2 is shaded, but the region enclosed by circle 1 is not. When we refer to the area of a circle, we mean the area of the enclosed circular region.

Lesson 2 Glossary Terms

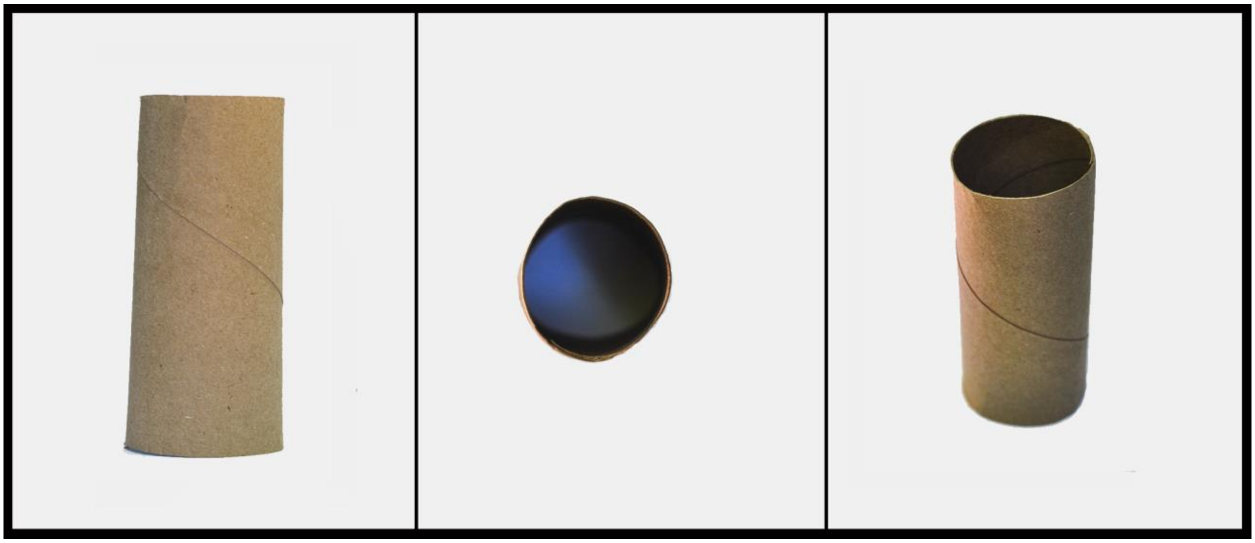
radius
diameter
circumference
circle

7.3.3 Exploring Circumference Date______________________

Student Objective: Explore the circumference of circles.

3.1: Which Is Greater?

Clare wonders if the height of the toilet paper tube or the distance around the tube is greater. What information would she need in order to solve the problem? How could she find this out?

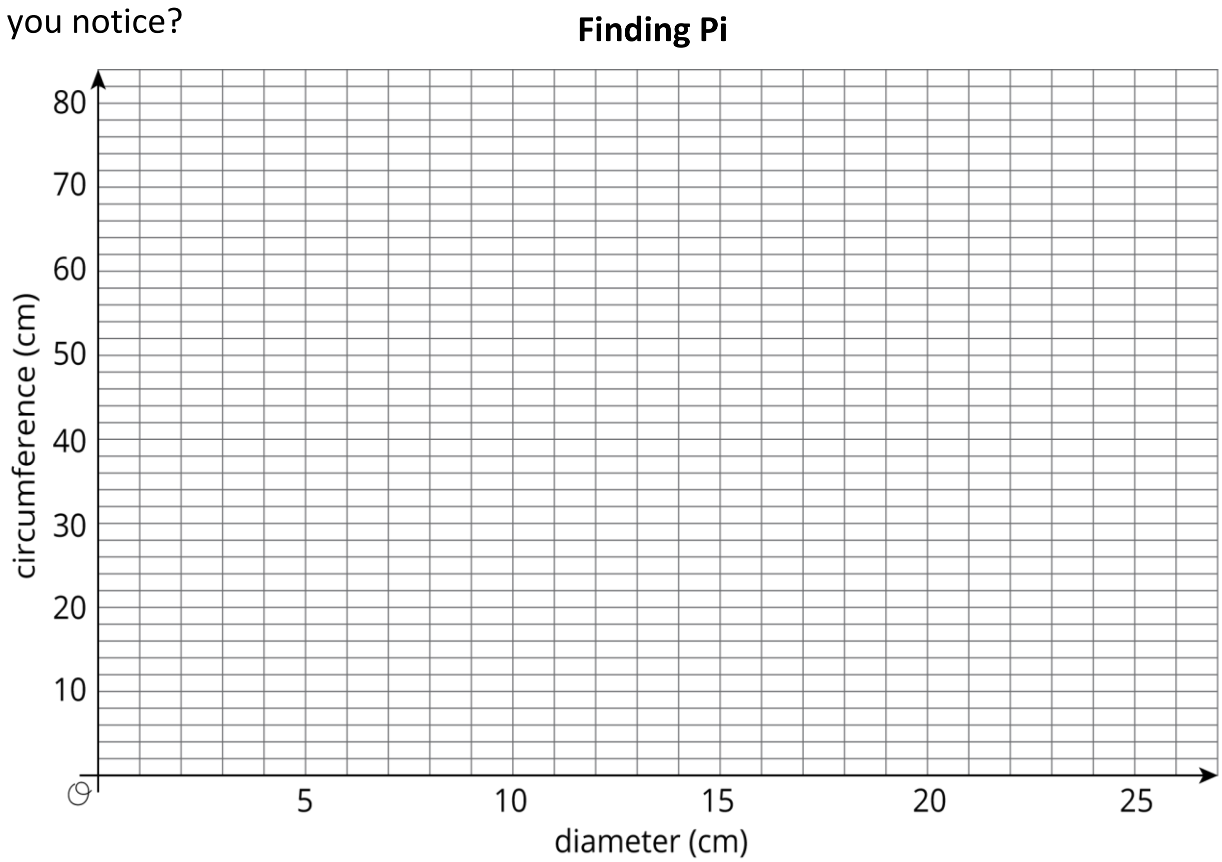


3.2: Measuring Circumference and Diameter

1. Measure the diameter and the circumference of the circle that your teacher gives you to the nearest tenth of a centimeter. Record your measurements in the table.

Object	diameter (cm)	Circumference (cm)	$\pi = \frac{Circumference}{diameter}$

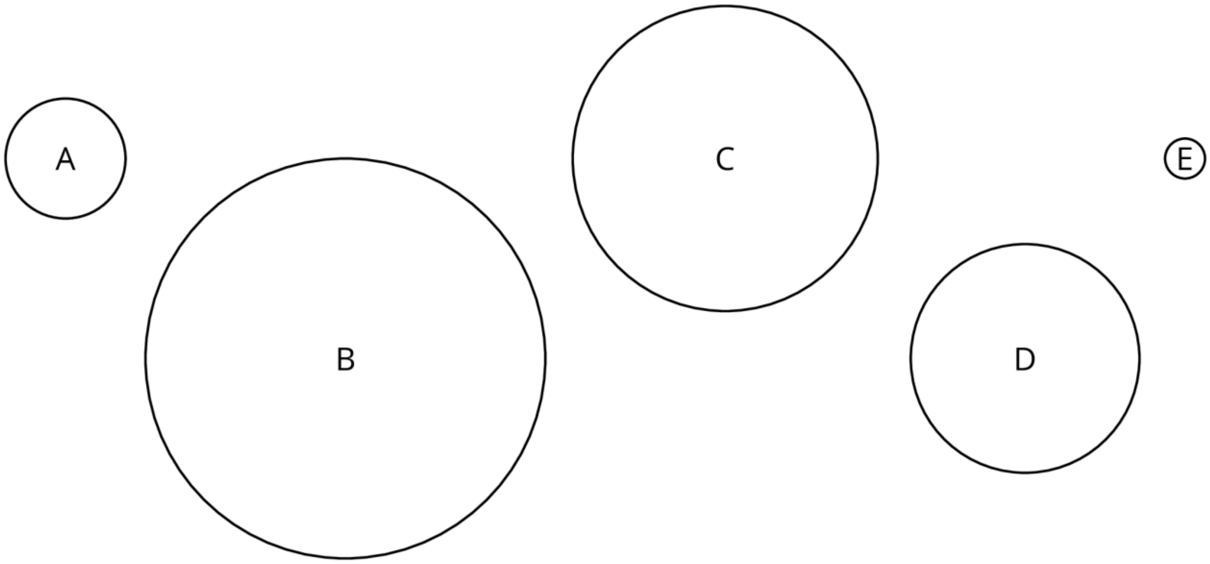
2. Plot the diameter and circumference values from the table on the coordinate plane. What do you notice?



3. We have an approximation of a straight line. The equation for this proportional relationship is $\pi = \frac{Circumference}{diameter}$. Find your approximation of π. ___________ Record it in the table. Find the approximation of π on the calculator.______________

3.3: Calculating Circumference and Diameter

Here are five circles. One measurement for each circle is given in the table.



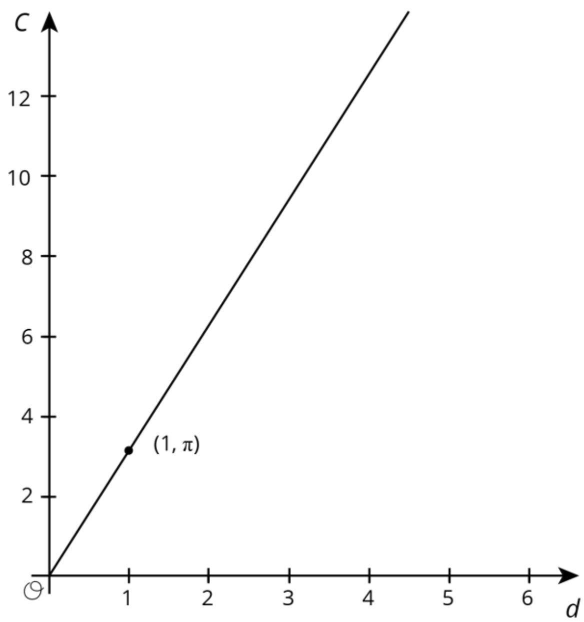
Use the equation for the proportional relationship, $C = \pi d$, to complete the table. Round to the same place as the given measurement.

	diameter (cm)	circumference (cm)
circle A	3	
circle B	10	
circle C		24
circle D		18
circle E	1	

Lesson 3 Summary

There is a proportional relationship between the diameter and circumference of any circle. That means that if we write C for circumference and d for diameter, we know that $C = kd$, where k is the constant of proportionality.

The exact value for the constant of proportionality is called **π**. Some frequently used approximations for π are $\frac{22}{7}$, 3.14, and 3.14159, but none of these is exactly π.



We can use this to estimate the circumference if we know the diameter, and vice versa. For example, using 3.1 as an approximation for π, if a circle has a diameter of 4 cm, then the circumference is about $(3.1) \cdot 4 = 12.4$ or 12.4 cm.

The relationship between the circumference and the diameter can be written as

$$C = \pi d$$

Lesson 3 Glossary Terms

pi (π)

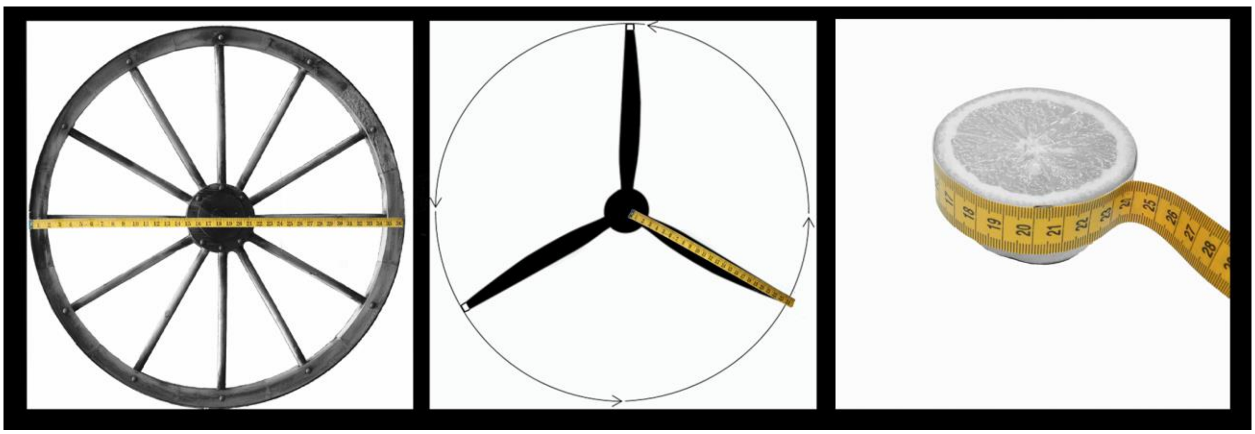
7.3.4 Applying Circumference Date______________________

Student Objective: Use π to solve problems.

4.1: What Do We Know? What Can We Estimate?

Here are some pictures of circular objects, with measurement tools shown. The measurement tool on each picture reads as follows:

- Wagon wheel: 3 feet
- Plane propeller: 24 inches
- Sliced Orange: 20 centimeters



1. For each picture, which measurement is shown?

2. Based on this information, what measurement(s) could you estimate for each picture?

4.2: Using π

In the previous activity, we looked at pictures of circular objects. One measurement for each object is listed in the table.

Your teacher will assign you an approximation of π to use for this activity.

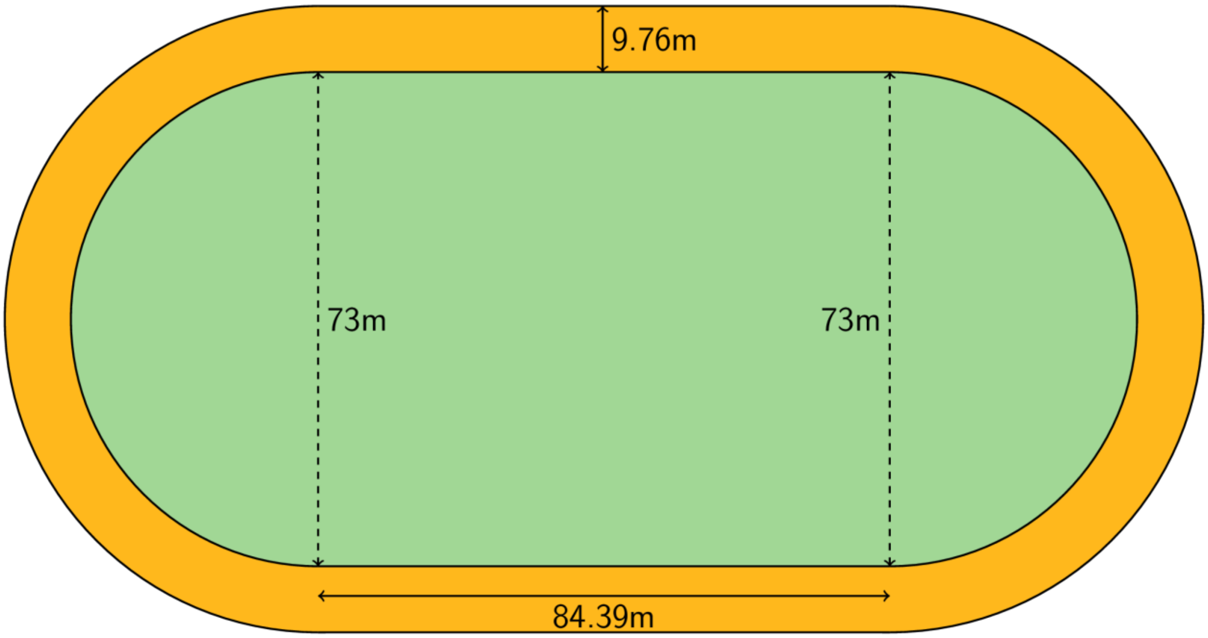
1. Complete the table.

object	radius	diameter	circumference
wagon wheel		3 ft	
airplane propeller	24 in		
orange slice			20 cm

2. A bug was sitting on the tip of the propeller blade when the propeller started to rotate. The bug held on for 5 rotations before flying away. How far did the bug travel before it flew off?

4.3: Around the Running Track

The field inside a running track is made up of a rectangle that is 84.39 m long and 73 m wide, together with a half-circle at each end.

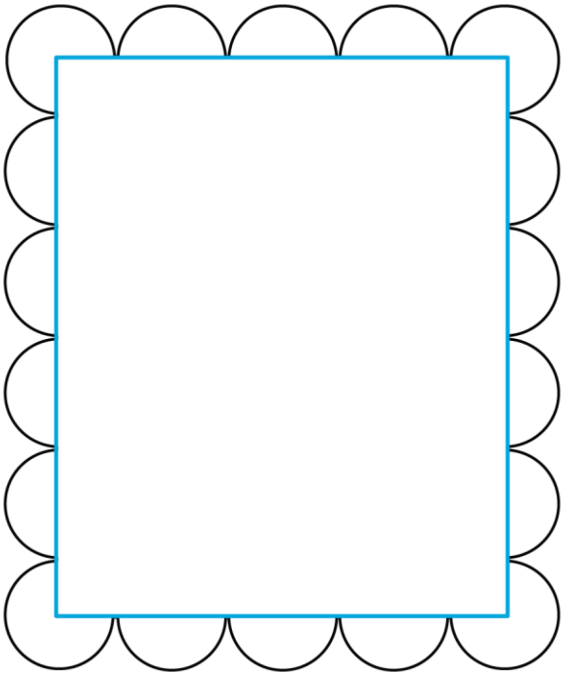


1. What is the distance around the inside of the track? Explain or show your reasoning.

2. The track is 9.76 m wide all the way around. What is the distance around the outside of the track? Explain or show your reasoning.

4.4: Measuring a Picture Frame

Kiran bent some wire around a rectangle to make a picture frame. The rectangle is 8 inches by 10 inches.



1. Find the perimeter of the wire picture frame. Explain or show your reasoning.

2. If the wire picture frame were stretched out to make one complete circle, what would its radius be?

Lesson 4 Summary

The circumference of a circle, C, is π times the diameter, d. The diameter is twice the radius, r. So if we know any one of these measurements for a particular circle, we can find the others. We can write the relationships between these different measures using equations:

$d = 2r$ $C = \pi d$ $C = 2\pi r$

If the diameter of a car tire is 60 cm, that means the radius is 30 cm and the circumference is $60 \cdot \pi$ or about 188 cm.

If the radius of a clock is 5 in, that means the diameter is 10 in, and the circumference is $10 \cdot \pi$ or about 31 in.

If a ring has a circumference of 44 mm, that means the diameter is $44 \div \pi$, which is about 14 mm, and the radius is about 7 mm.

7.3.6 Estimating Area Date______________________

Student Objective: Estimate the areas of weird shapes.

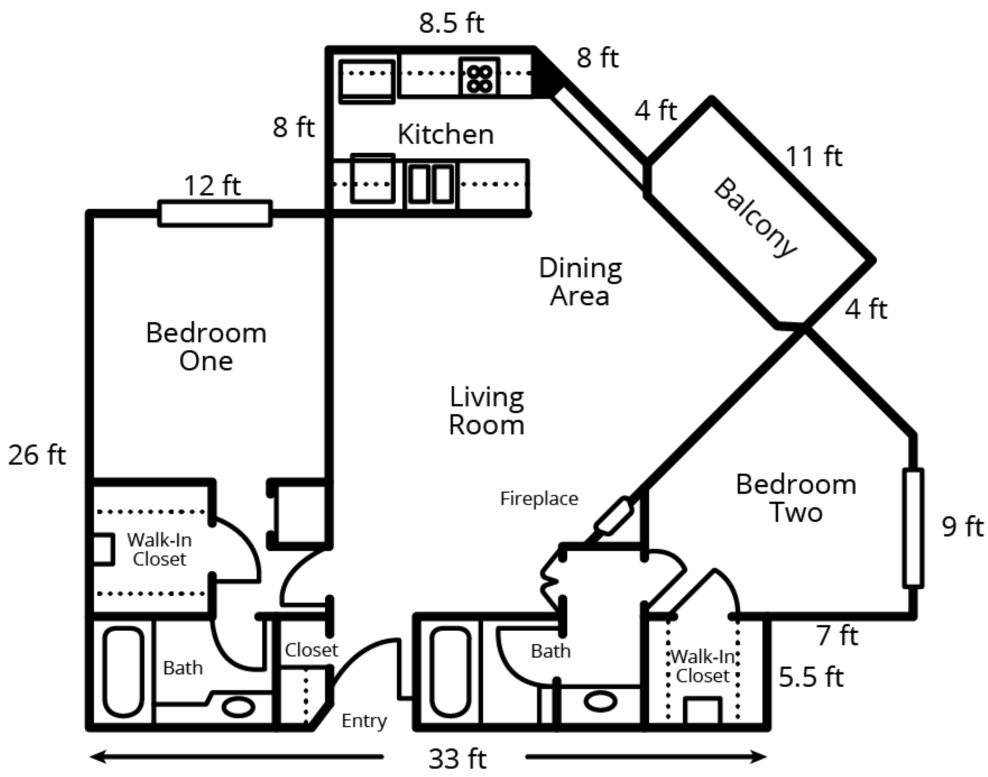
6.1: Mental Calculations

Find a strategy to estimate each calculation mentally:

1. $599 + 87$
2. $254 - 88$
3. $99 \cdot 75$

6.2: House Floorplan

Here is a floor plan of a house. Approximate lengths of the walls are given.



What is the approximate area of the home, including the balcony? Explain or show your reasoning.

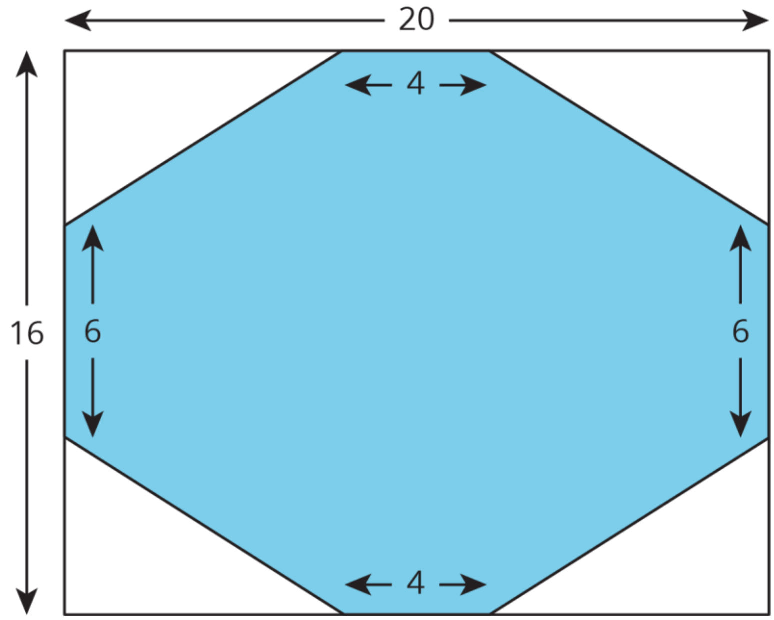
6.3: Area of Nevada

Estimate the area of Nevada in square miles. Explain or show your reasoning.

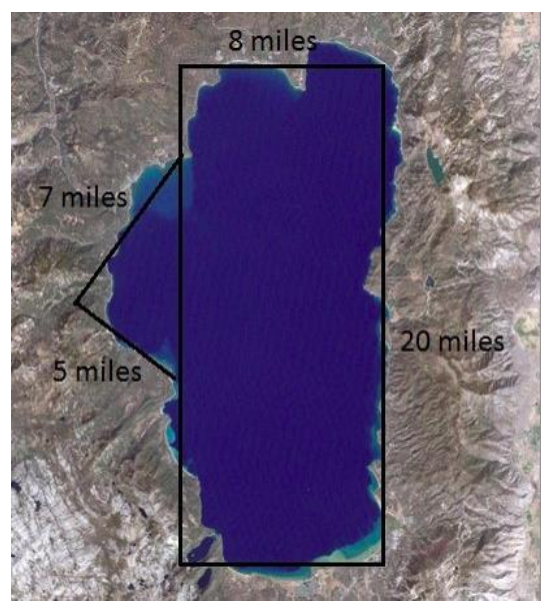
Lesson 6 Summary

We can find the area of some complex polygons by surrounding them with a simple polygon like a rectangle. For example, this octagon is contained in a rectangle.

The rectangle is 20 units long and 16 units wide, so its area is 320 square units. To get the area of the octagon, we need to subtract the areas of the four right triangles in the corners. These triangles are each 8 units long and 5 units wide, so they each have an area of 20 square units. The area of the octagon is $320 - (4 \cdot 20)$ or 240 square units.



We can estimate the area of irregular shapes by approximating them with a polygon and finding the area of the polygon. For example, here is a satellite picture of Lake Tahoe with some one-dimensional measurements around the lake.



The area of the rectangle is 160 square miles, and the area of the triangle is 17.5 square miles for a total of 177.5 square miles. We recognize that this is an approximation, and not likely the exact area of the lake.

7.3.7 Exploring the Area of a Circle Date______________________

Student Objective: Investigate the areas of circles.

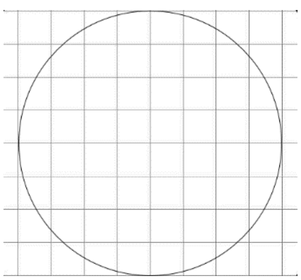
7.1: Estimating Areas

Your teacher will show you some figures. Decide which figure has the largest area. Be prepared to explain your reasoning.

7.2: Estimating Areas of Circles

Your teacher will give your group two circles of different sizes.

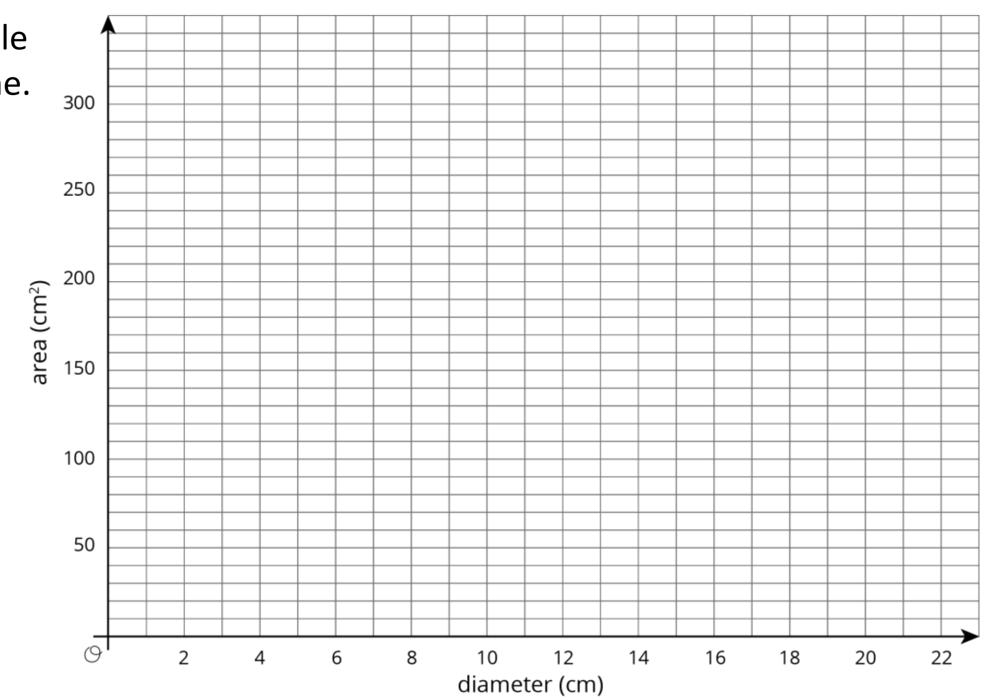
1. For each circle, use the squares on the graph paper to measure the diameter and estimate the **area of the circle**. Record your measurements in the table.



diameter (cm)	estimated area (cm^2)

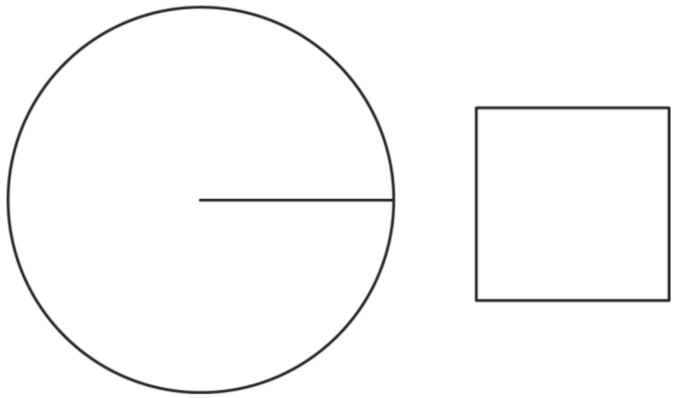
2. Plot the values from the table on the class coordinate plane. Then plot the class's data points on your coordinate plane.

3. Is this the graph of a proportional relationship? Explain.



7.3: Covering a Circle

Here is a square whose side length is the same as the radius of the circle.



How many of the squares do you think it would take to cover the circle exactly?

Lesson 7 Summary

The circumference C of a circle is proportional to the diameter d, and we can write this relationship as $C = \pi d$. The circumference is also proportional to the radius of the circle, and the constant of proportionality is $2 \cdot \pi$ because the diameter is twice as long as the radius. However, the **area of a circle** is *not* proportional to the diameter (or the radius).

The area of a circle with radius r is a little more than 3 times the area of a square with side r so the area of a circle of radius r is approximately $3r^2$. We saw earlier that the circumference of a circle of radius r is $2\pi r$. If we write C for the circumference of a circle, this proportional relationship can be written $C = 2\pi r$.

The area A of a circle with radius r is approximately $3r^2$. Unlike the circumference, the area is not proportional to the radius because $3r^2$ cannot be written in the form kr for a number k. We will investigate and refine the relationship between the area and the radius of a circle in future lessons.

Lesson 7 Glossary Terms

area of a circle

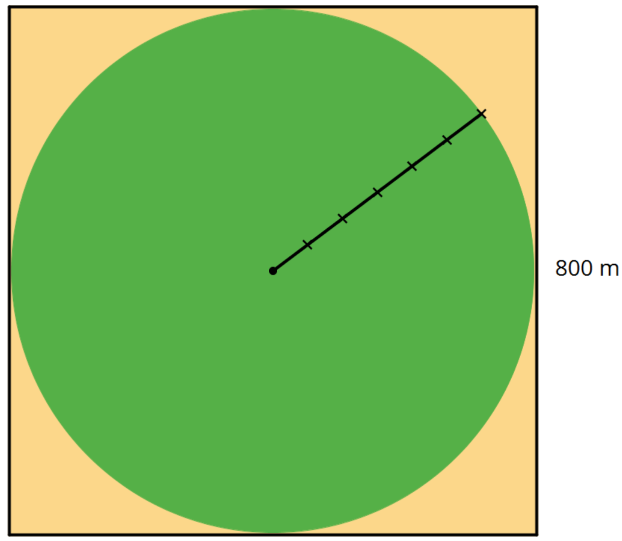
7.3.8 Relating Area to Circumference Date______________________

Student Objective: Rearrange circles to calculate their areas.

8.1: Irrigating a Field

A circular field is set into a square with an 800 m side length. Estimate the field's area.

a. About 5,000 m^2
b. About 50,000 m^2
c. About 500,000 m^2
d. About 5,000,000 m^2
e. About 50,000,000 m^2



8.2: Making a Polygon out of a Circle

Your teacher will give you a circular object, a marker, and paper of a different color.

- Follow these instructions to create a visual display:
- Using a thick marker, trace your circle in two separate places on the same piece of paper.
- Cut out both circles, cutting around the marker line.
- Fold and cut one of the circles into fourths.
- Arrange the fourths so that straight sides are next to each other, but the curved edges are alternately on top and on bottom. Pause here so your teacher can review your work.
- Fold and cut the fourths in half to make eighths. Arrange the eighths next to each other, like you did with the fourths.
- Glue the remaining circle and the new shape onto a piece of paper that is a different color.

After you finish gluing your shapes, answer the following questions.

a. The areas of the two shapes are _______________________.

b. The shape made of the circle pieces looks like a ________________________.

c. How could you find the area of this polygon?

Area of a parallelogram =______________x_______________
Area of circle = ______________x______________
Area$_\circ$ =

8.4: Tiling a Table

Elena wants to tile the top of a circular table. The diameter of the table top is 28 inches. What is its area?

Lesson 8 Summary

If C is a circle's circumference and r is its radius, then $C = 2\pi r$. The area of a circle can be found by taking the product of half the circumference and the radius.

If A is the area of the circle, this gives the equation:

$$A = \frac{1}{2}(2\pi r) \cdot r$$

This equation can be rewritten as:

$$A = \pi r^2$$

This means that if we know the radius, we can find the area. For example, if a circle has radius 10 cm, then the area is about $(3.14) \cdot 100$ which is 314 cm^2.

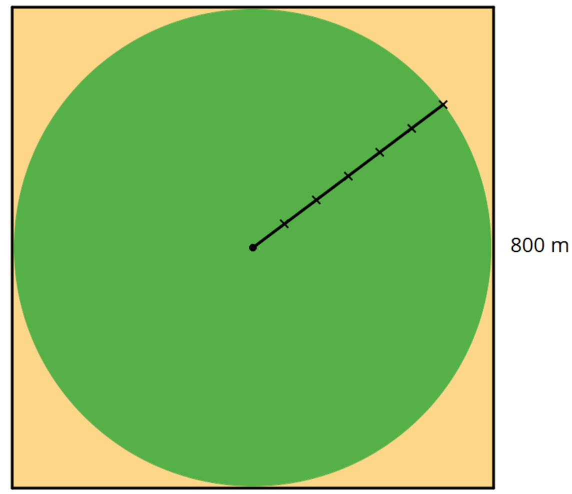
If we know the diameter, we can figure out the radius, and then we can find the area. For example, if a circle has a diameter of 30 ft, then the radius is 15 ft, and the area is about $(3.14) \cdot 225$ which is approximately 707 ft^2.

7.3.9 Applying Area of Circles Date______________________

Student Objective: Find the areas of shapes made up of circles.

9.1: Still Irrigating the Field

The area of this field is about 500,000 m^2. What is the area of the circular field in terms of π? ______________________ What is the field's area to the nearest square meter? Assume that the side lengths of the square are exactly 800 m.502,400 m^2



a. 502,640 m^2

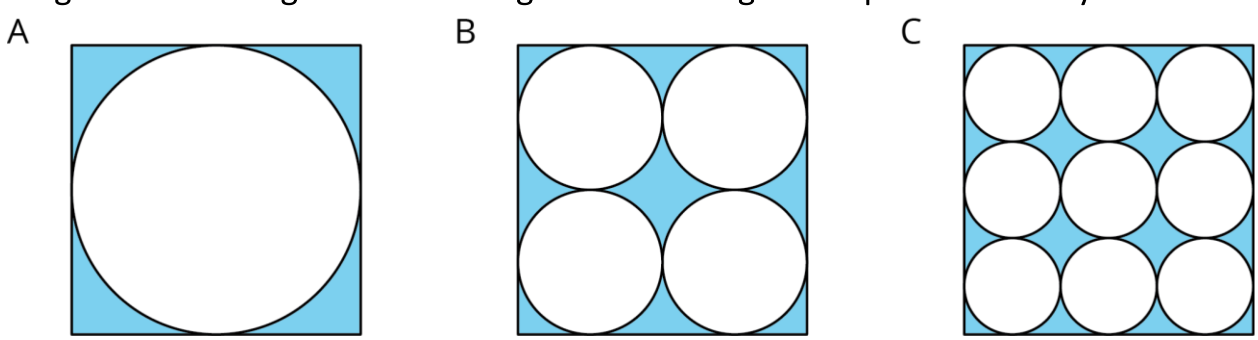
b. 502,655 m^2

c. 502,656 m^2

d. 502,857 m^2

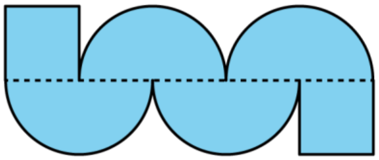
9.2: Comparing Areas Made of Circles

1. Each square has a side length of 12 units. Compare the areas of the shaded regions in the 3 figures. Which figure has the largest shaded region? Explain or show your reasoning.

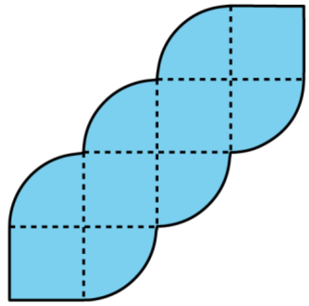


2. Each square in Figures D and E has a side length of 1 unit. Compare the area of the two figures. Which figure has more area? How much more? Explain or show your reasoning.

D

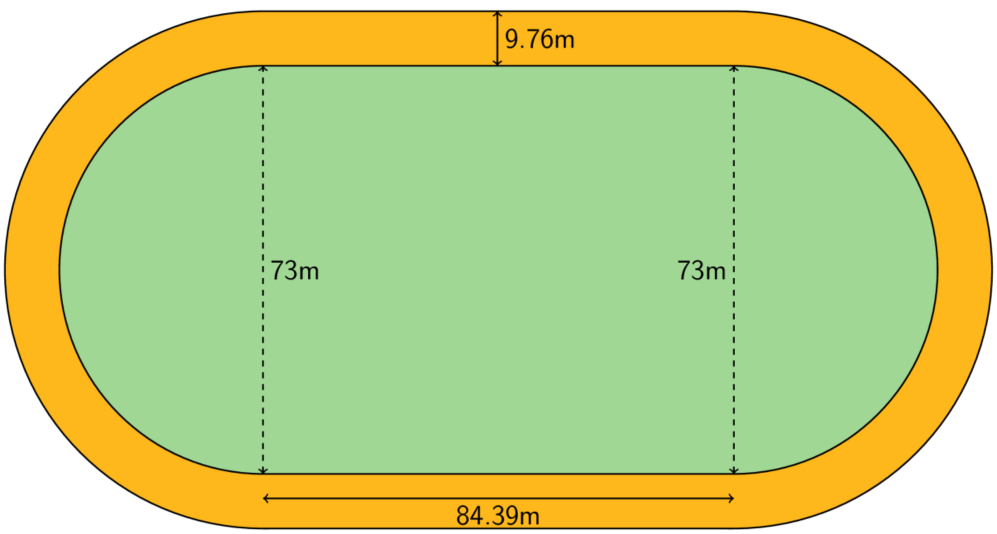


E



9.3: The Running Track Revisited

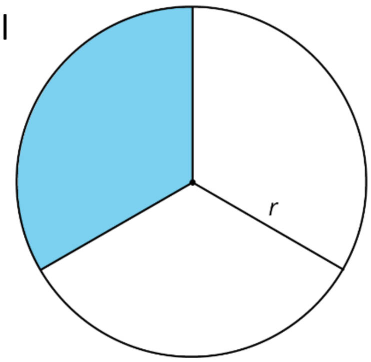
The field inside a running track is made up of a rectangle 84.39 m long and 73 m wide, together with a half-circle at each end. The running lanes are 9.76 m wide all the way around.



What is the area of the running track that goes around the field? Explain or show your reasoning.

Lesson 9 Summary

The relationship between A, the area of a circle, and r, its radius, is $A = \pi r^2$. We can use this to find the area of a circle if we know the radius. For example, if a circle has a radius of 10 cm, then the area is $\pi \cdot 10^2$ or 100π cm^2. We can also use the formula to find the radius of a circle if we know the area. For example, if a circle has an area of 49π m^2 then its radius is 7 m and its diameter is 14 m.



Sometimes instead of leaving π in expressions for the area, a numerical approximation can be helpful. For the examples above, a circle of radius 10 cm has area about 314 cm^2. In a similar way, a circle with area 154 m^2 has radius about 7 m.

We can also figure out the area of a fraction of a circle. For example, the figure shows a circle divided into 3 pieces of equal area. The shaded part has an area of $\frac{1}{3}\pi r^2$.

7.3.10 Distinguishing Area and Circumference Date______________________

Student Objective: Contrast circumference and area.

10.1: Filling the Plate

About how many cheese puffs can fit on the plate in a single layer? Be prepared to explain your reasoning.

10.2: Card Sort: Circle Problems

Your teacher will give you cards with questions about circles.

1. Sort the cards into two groups based on whether you would use the circumference or the area of the circle to answer the question. Pause here so your teacher can review your work.

2. Your teacher will assign you a card to examine more closely. What additional information would you need in order to answer the question on your card?

3. Estimate measurements for the circle on your card.

4. Use your estimates to calculate the answer to the question.

10.4: Analyzing Circle Claims

Here are two students' answers for each question. Do you agree with either of them? Explain or show your reasoning.

1. How many feet are traveled by a person riding once around the merry-go-round?

- Clare says, "The radius of the merry-go-round is about 4 feet, so the distance around the edge is about 8π feet."
- Andre says, "The diameter of the merry-go-round is about 4 feet, so the distance around the edge is about 4π feet."

2. How much room is there to spread frosting on the cookie?

- Clare says "The radius of the cookie is about 3 centimeters, so the space for frosting is about 6π cm^2."
- Andre says "The diameter of the cookie is about 3 inches, so the space for frosting is about 2.25π in^2."

3. How far does the unicycle move when the wheel makes 5 full rotations?

- Clare says, "The diameter of the unicycle wheel is about 0.5 meters. In 5 complete rotations it will go about $\frac{5}{2}\pi$ m^2."

- Andre says, "I agree with Clare's estimate of the diameter, but that means the unicycle will go about $\frac{5}{4}\pi$ m."

Lesson 10 Summary

Sometimes we need to find the circumference of a circle, and sometimes we need to find the area. Here are some examples of quantities related to the circumference of a circle:

- The length of a circular path.
- The distance a wheel will travel after one complete rotation.
- The length of a piece of rope coiled in a circle.

Here are some examples of quantities related to the area of a circle:

- The amount of land that is cultivated on a circular field.
- The amount of frosting needed to cover the top of a round cake.
- The number of tiles needed to cover a round table.

In both cases, the radius (or diameter) of the circle is all that is needed to make the calculation. The circumference of a circle with radius r is $2\pi r$ while its area is πr^2. The circumference is measured in linear units (such as cm, in, km) while the area is measured in square units (such as cm^2, in^2, km^2).